STRUCTURAL PARAMETER ESTIMATION BIAS IN WELFARE EFFECTS OF TAX POLICY

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ABSTRACT

A change in income tax policy can result in welfare loss to households and consequently reduce their consumption level. The government therefore gains benefit from welfare analysis and estimation to investigate the impact of its policy. However, welfare estimates are sensitive to calibrated or estimated parameter values. Significant biases in structural parameter estimates may lead to biases in welfare estimates and subsequently affect policy conclusions. Using a simple RBC model, we investigate the relationship between the bias in welfare cost estimates and the biases in structural parameter estimates and find the bias varies nonlinearly over the parameter space. Furthermore, the bias in welfare estimates depends upon the bias in different calibrated parameter values in very different ways. In our simple model, for example, bias in welfare estimation is increased if we assume too high a depreciation rate of capital or too low a capital share.

Keywords: Welfare Analysis, Welfare Estimates, Income Tax Policy, Structural Parameter Estimation Bias.

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Introduction

Studying the effect on welfare of different policy experiments is an important function of Dynamic Stochastic General Equilibrium (DSGE) models. Such Micro-founded models can be used, for example, to abstractly explain the behaviour of the economy and the effect of alternative policies upon welfare (Judd, 1987; Heer, 2003; Domeji & Heathcote, 2004; Mukoyama, 2010). Policy implications are then based on the estimated welfare cost expressed as a function of estimated structural parameters specified in DSGE models. As a consequence, any bias in structural parameter estimates naturally leads to the bias in welfare cost estimates, in turn leading to potentially misleading policy implications. In this paper, we illustrate the relationship between parameter and welfare cost estimation bias under a simple Real Business Cycle (RBC) framework. We make suggestions on how to calibrate and estimate structural parameters in order to ultimately reduce the bias in welfare cost estimates within this framework.

In previous work, Dolmas (1998), Tallarini (2000) and Lagos and Wright (2005) demonstrate the sensitivity of welfare estimates to structural parameter values in DSGE models. They show that welfare cost estimates can be varied substantially according to choices of parameter values specified in the model. However, they do not take parameter uncertainty into consideration. Levin, et al. (2006) and Pablo (2007) study the role of parameter uncertainty upon welfare analysis using a Bayesian approach. They estimate the posterior distribution of welfare cost to study the effect of parameter uncertainty on the estimated welfare cost of inflation. The results suggest that parameter uncertainty has a significant effect on the welfare estimates and can lead to the possibility of unreasonable or misleading results. In this paper, our contribution is to show how the biases in structural parameter estimates affect the bias in welfare cost estimates. Unlike Dolmas (1998), Tallarini (2000) and Lagos and Wright (2005), we do not assume all the parameters are known and, in contrast to Levin et al. (2006) and Pablo (2007), we consider the role of estimation bias upon welfare analysis rather than the role of parameter uncertainty.

In the case of a simple RBC framework, the structural parameters influencing households' intratemporal decisions, such as the utility of leisure, capital share and depreciation rate of capital, play an important role in quantifying the welfare of households. Hence, estimating these parameters raises concerns for policy makers, especially when these parameters are converted into other quantities of interest such as welfare cost of alternative fiscal policies. The current challenges in the estimation of DSGE models and consequences to the structural parameter estimation have been discussed in detail by Schorfheide (2011). Other studies which examine related problems include Canova and Sala (2009) who consider identification issues and Ruge-Murcia (2007) discuss small sample properties. In this paper, we therefore set our model specification to explore the impact from small sample bias and parameter identification problems of key structural parameters to welfare cost estimation.

We estimate structural parameters in a simple RBC model using Maximum Likelihood Estimation (MLE) with artificial data sets and carry out a quantitative analysis of the welfare cost of fiscal policy changes. We design Monte Carlo experiments to compute the bias in estimation of both structural parameters and welfare costs under different fiscal schemes. In our policy experiments, we take the estimated RBC model as a representation of the economy and consider alternative values of the income tax rate to explore the impact of different fiscal policies upon welfare. To avoid the dependency of welfare cost estimates on the functional form of utility and any model misspecification, we assume we know true functional forms of both utility and trend specification in the model.[1] Finally, we estimate Response Profiles to evaluate how the impact of structural parameter estimation bias on welfare changes over the parameter space and across alternative tax policies.

From the Response Profiles of bias in welfare cost estimates expressed as functions of biases in structural parameter estimators, we can make a number of conclusions. First, this relationship is not linear and the bias in welfare cost estimates responds very differently to the bias of different structural parameter estimates. For example, calibrating too high a depreciation rate of capital, and thus introducing a positive bias in this parameter, creates a large negative bias in welfare estimates. On the other hand, the bias in welfare estimates is relatively small

[1] This is important as Otrok (2001) states that the welfare cost can be made as large as one wishes by changing the functional form of utility. Wincoop (1999) investigates alternative stochastic endowment processes and finds that the welfare estimates also depend on the stochastic process specified in the model. A stochastic process which propagates a shock over time (i.e. AR(1) process in growth rates) will introduce a high cost associated with the elimination of the fluctuations in consumption.

when we calibrate too high a capital share. Second, the identification of the key structural parameters is important. Due to the complexity and nonlinearity of the state space representation of the underlying DSGE model, not all the structural parameters are generally identified. As macroeconomists face a problem of parameter identification, they tend to calibrate or fix some parameters by, for example, using micro-evidence or a priori selection while estimating the rest. However, calibrating a value of a parameter too far from the unknown true value leads to a serious bias in other remaining parameter estimators and, more importantly, in welfare cost estimates. In our model, the capital share and the depreciation rate of capital are partially identified. Thus we have the choice to calibrate one and estimate the other. This choice, it turns out, matters for the bias in welfare cost estimates. Finally, the structural parameter estimation bias exacerbates the bias in welfare cost estimates due to the nonlinearity in the welfare cost function. Thus bias in structural parameter estimates subsequently leads to a reduction in the accuracy of estimated welfare cost and mislead policy advices. In this particular exercise, we find that calibrating the depreciation rate of capital too high, or the capital share too low, leads to a larger bias in welfare estimates. As a result, we can reduce the bias in welfare cost estimates by calibrating the depreciation rate of capital in a range of low values, or the capital share in a range of high values.

A Real Business Cycle Model

The purpose of this study is to illustrate the potential bias in structural parameter estimates induced by the small sample bias and parameter identification problem. To maintain a tractable analysis, as a result, we consider a standard and small-sized RBC model that is a simple variation of Hansen (1985) as the Data Generating Process (DGP) used in our experiments.

The problem of the households can be written as follows.

$$\max \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \{U(C_t) + B_t A(1 - H_t)\}$$

subject to

$$C_t + I_t = (1 - \tau)(R_t K_t + W_t H_t) \quad (1)$$

$$I_t = K_{t+1} - (1 - \delta) K_t \quad (2)$$

$$\ln(B_{t+1}) = \rho_B \ln(B_t) + \epsilon_{B,t+1} \quad (3)$$

where $\beta \in (0,1)$ is a discount factor, $A > 0$ is a utility of one unit of leisure, $\delta \in (0,1)$ is a depreciation rate of capitaland B_t is a stationary AR(1) exogenous process of the preference shock governed by a measure of a persistence ρ_B and a Gaussian shock $\epsilon_{B,t} \sim N(0, \sigma_B^2)$.

Households in this economy optimise their expected discounted life-time utility by choosing each period consumption (C_t)), hours worked (H_t) and next-period capital holding (K_{t+1}) subject to their budget constraint (1), a capital accumulation equation (2) and a stationary AR(1) exogenous process of the preference shock (B_t) to households' labor supply (3). The endowment of time is normalised to be 1 which can be taken as leisure ($1 - H_t$) or used as hours worked (H_t). The sources of income for households are from supplying capital and labour services to the firms. Income in this setting is taxed at the rate of τ. The after-tax income can then be either consumed or invested. Let $I_t$be investment, $R_t$be the rental rate of capital and W_t be the wage rate at period t.

The values of the structural parameters specified in this problem govern the decision of the households. The discount factor (β) influences the intertemporal trade-off between the consumption this period and the consumption next period. A low discount factor implies that the future consumption is highly discounted and households prefer consuming more today. The intratemporal decisions, on the other hand, are governed by the utility of leisure (A) and the depreciation rate of capital (δ). The utility of leisure affects the trade-off between the consumption and the leisure this period, while the depreciation rate of capital (δ) governs how households allocate their after-tax income between the consumption and capital holdings. When the depreciation rate is low, households have more incentive to invest in long-lived capital goods and allocate less of their after-tax income to consumption goods. As we assume the preference persistence (ρ_B) is between 0 and 1, the preference shock to households' labor supply is transitory.

Tax revenue collected from households is used to finance exogenous government spending in which the government budget constraint is given by

$$\tau(\mathrm{R_t} K_t + W_t H_t) = G_t$$

We can write the problem for firms as

$$\max \{Y_t - R_t K_t - W_t H_t\}$$

subject to

$$Y_t = K_t^{\alpha}(z_t H_t)^{1-\alpha} \quad (4)$$

$$\ln(\mathrm{Z_{t+1}}) = \rho_Z \ln(Z_t) + \epsilon_{Z,t+1} \quad (5)$$

where $\alpha \in (0,1)$ is a capital share and Z_t is a stationary AR(1) exogenous process of the technology shock governed by a measure of a persistence ρ_Z and a Gaussian shock $\epsilon_{Z,t} \sim N(0, \sigma_Z^2)$.

The firms maximise their profit subject to the labor-augmenting Cobb-Douglas production function using capital and labor as inputs (4) and a stationary AR(1) technology (Z_t) process (5). Here the revenue is obtained by selling goods, denoted by Y_t, to the households while the costs are incurred from renting households' capital and labor services. In this problem, the capital share (α) represents the share of total output paid to capital services. The technology shock to the production function is assumed to be transitory as the technology persistence (ρ_Z) is set between 0 and 1.

The First Order Conditions (FOCs) for households' utility maximisation and firms' profit maximisation problems are as follows.

$$U'(C_t) = \beta \mathbb{E}_t\{U'(C_{t+1})((1 - \tau)R_{t+1} + 1 - \delta)\} \quad (6)$$

$$U'(C_t) = \frac{B_t A}{(1-\tau)W_t} \quad (7)$$

$$R_t = \alpha K_t^{\alpha-1}(z_t H_t)^{1-\alpha} \quad (8)$$

$$W_t = (1-\alpha) z_t^{1-\alpha} K_t^{\alpha} H_t^{-\alpha} \quad (9)$$

These necessary conditions characterise the equilibrium decision rules for the households and firms. Equation (6) is an Euler equation for consumption stating that the marginal rate of substitution between the consumption at period t and the consumption at period $t+1$ equals the after-tax return of capital. Equation (7) is a labor supply equation stating that the marginal rate of substitution between consumption and leisure must equal to the after-tax wage rate. Equations(8) and (9) come from the firms' problem implying that the rental rate of capital and wage rate are set equal to the marginal productivity of an additional capital and labor respectively.

In the equilibrium, households will choose allocations of $\{C_t, H_t, K_{t+1}\}_{t=0}^{\infty}$ whereas firms will choose allocations of $\{K_t, H_t\}_{t=0}^{\infty}$ such that, given a sequence of prices $\{W_t, R_t\}_{t=0}^{\infty}$ and a tax policy $\{\tau\}$, households and firms optimise their utility and profit respectively, government's budget constrain is satisfied and all markets clear, or equivalently in equilibrium such that $Y_t = C_t + I_t + G_t$.

We define $\Omega \equiv \{\rho_Z, \sigma_Z, \rho_B, \sigma_B, \beta, A, \alpha, \delta\}$ as a set of structural parameters. We then have all steady-state variables as a function of structural parameters and the tax policy; $C_*(\Omega;\tau)$, $H_*(\Omega;\tau)$, $K_*(\Omega;\tau)$,$H_*(\Omega;\tau)$. By deriving a state-space representation from FOCs, the model can then be estimated using the Maximum Likelihood Estimation (MLE). We denote a set of estimated structural parameters as $\widehat{\Omega}$.

Welfare Cost Calculation

Given the estimated RBC model, we compute the deterministic welfare cost estimate by calculating how much consumption in the steady state households are willing to give up in order to be indifferent between two economies with different levels of income tax rate. This is the well-known concept of compensation variation.

At the income tax rate of τ_*, we can write the expected life-time welfare of households at the steady state as

$$(1-\hat{\beta})W(\widehat{\Omega};\tau_*) = U\left(C_*(\widehat{\Omega};\tau_*)\right) + \hat{A}(1 - H_*(\widehat{\Omega};\tau_*)).$$

If there is a deviation in the income tax rate from τ_* to τ, households' consumption at the steady state changes by a factor Δ of the initial consumption level and hours worked reaches a new steady-state level, denoted by $H_s(\widehat{\Omega};\tau)$. The expected life-time welfare of households under this new tax policy can be written as

$$(1-\hat{\beta})W_\Delta(\widehat{\Omega};\tau) = U\left(C_*(\widehat{\Omega};\tau_*)\Delta\right) + \hat{A}(1 - H_s(\widehat{\Omega};\tau)).$$

We measure the welfare cost of a change in the income tax rate as the value of Δ which solves $W(\widehat{\Omega};\tau_*) = W_\Delta(\widehat{\Omega};\tau)$. The property of the deterministic welfare cost is illustrated in Figure 1 and we can interpret the value of Δ as follows. When there is a decrease in the income tax rate from τ_* to τ ($\tau_* > \tau$), households benefit from this policy change and are willing to give up $1-\Delta$ percent of their initial consumption to stay under this policy. On the other hand, if there is an increase in the income tax rate ($\tau_* < \tau$), households suffer from a welfare loss and require compensation of $1-\Delta$ of their initial consumption to stay under the new tax rate.

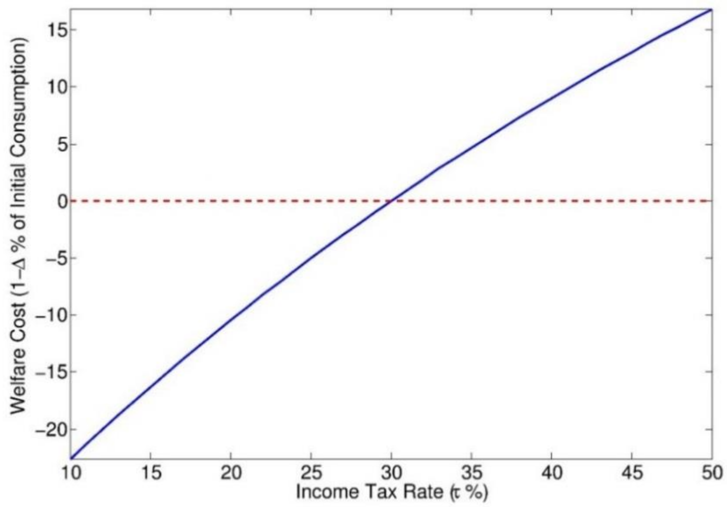
By solving for the analytical solution for the welfare cost estimate, we obtain

$$W(\widehat{\Omega};\tau_*) = W_\Delta(\widehat{\Omega};\tau)$$

$$\Delta((\widehat{\Omega};\tau_*,\tau) = exp\left\{\frac{\upsilon(1-\hat{\alpha})(\tau_* - \tau)}{(\upsilon-(1-\tau)\kappa)(\upsilon-(1-\tau_*)\kappa)}\right\} \quad (10)$$

where $\kappa = \hat{\alpha}\hat{\delta}\hat{\beta}$ and $\upsilon = 1-\hat{\beta}+\hat{\beta}\hat{\delta}$.

Here the welfare cost estimate is expressed as a nonlinear function of estimated structural parameters and the two income tax rates of interest. Hence, the magnitude of estimated welfare cost depends on the values of these structural parameter estimates. Intuitively, for example, when the depreciation rate is high, households have less incentive to invest in short-lived capital goods and allocate more of their after-tax income to consumption goods. Any change in the income tax rate therefore changes consumption levels more than the one when the depreciation rate is high. In this situation, the changes in consumption are costly and imply a larger welfare cost of fiscal policy. This is similar to the case when the capital share is low as households have less incentive to invest in low-return capital goods. Any bias in structural parameter estimates may therefore induce the bias in welfare cost estimate and impact the results of policy experiments.



Notes: The baseline fiscal policy is $\tau_* = 30\%$.

Fig. 1: Welfare Cost of a Variation in Income Tax Rates (τ)

Estimation Issues

Canova and Sala (2009) and Schorfheide (2011) consider the difficulties in the estimation routine of DSGE models and the consequences for parameter estimation. As the welfare cost estimates are sensitive to the values of estimated structural parameters, the estimation issues have important implications for welfare analysis. In this paper, we investigate the importance of small sample bias and parameter identification for welfare analysis. We do not consider the issue of misspecification and failure in shock identification as we assume that the estimated model is the same as the true model we use to generate the artificial data set. To fix ideas, we provide a few comments on these estimation issues below.

Small Sample Biases. Data sets typically used in macroeconomics tend to be relatively small. Even when using consistent estimators, sample sizes are often too small for the estimators to be a useful approximation of the asymptotic estimators.

Parameter Identification Problem. DSGE Models tend to suffer from parameter identification issues that cause a reduction in our ability to draw inference about key structural parameters specified in the model. Kim (2003), Beyer and Farmer (2004) and Iskrev (2010), for example, provide cases of the unidentified DSGE models. There are many sources of parameter nonidentification which can be defined as follows. Observational equivalence occurs when two (or more) values of the parameters, $\Omega$and Ω^*, give the same likelihood value for all data sets. That is, $L(\Omega) = L(\Omega^*)$for all y. One example of observational equivalence is when some structural parameters are separable unrecoverable. By simultaneously changing these parameters by the right amount, the likelihood function is roughly unchanged and has the same height across some range of the parameter values. We refer these parameters as partially identified parameters. In addition, as the solution to the DSGE models tend to be in a log-linearised form, some structural parameters may disappear from the solution. The likelihood function in this case is noninformative over these parameters. As a result, the parameters are not identified and the parameter values cannot be estimated from the likelihood maximisation routine without any additional information.

Monte Carlo Experiments

We design Monte Carlo experiments to examine the implications of estimation issues in DSGE models and approximate the size of biases of the structural parameter and welfare cost estimation. The Monte Carlo experiments is as follows. Given the framework defined in Section 2, we have $\Omega \equiv \{\rho_Z, \sigma_Z, \rho_B, \sigma_B, \beta, A, \alpha, \delta\}$ as a set of true structural parameters. As the deterministic welfare cost estimate is derived as a function of behavioural parameter estimates, our focus in this paper is then on the group of parameters governing agents' behavior and the ones that suffer from identification issues and, in turn, are difficult to estimate. This group contains the utility of leisure (A), the capital share (α) and the depreciation rate of capital (δ).The range of true structural parameter values we consider for the DGP is as follows. We investigate a sensible range of the capital share $\alpha \in$ [0.2,0.4] and the depreciation rate of capital $\delta \in$ [0.005,0.05] in which cover common calibrated and estimated values in the literature. We then fix the remaining parameters as A = 3, β = 0.99, ρ_z = 0.95. σ_z = 0.007, ρ_B = 0.8 and σ_B = 0.007.

Denote a set of true parameter values in an experiment $i = \{1,2,\ldots,M\}$ by Ω^i. For each value of Ω^i, we compute the true welfare cost, denoted by $1 - \Delta(\Omega^i; \tau_*, \tau)$, when there is a change in the income tax rate τ = {10%, 20%, 30%, 40%, 50%} with the baseline fiscal policy of τ_* = 30%. Next, given a set of true parameter values Ω^i, we generate a data set of $y_t \equiv \{C_t(\Omega^i), H_t(\Omega^i)\}_{t=1}^{T}$ and estimate the structural parameters $\widehat{\Omega}^i$ via MLE. We replicate this process N times.

To explore the impact of small sample biases, we simulate the artificial data sets with a different sample size of $T = \{100, 200, 1000\}$, which are equivalent to 25-, 50- and 250-years of quarterly data. The problem of parameter identification under this framework can be seen through the relationship between the capital share and the depreciation rate where they are only partially identifiable. One way to solve the partial identification issue is to fix or calibrate one of the parameters and estimate others. We thus question how we should fix one of the parameters and how sensitive other related estimates, especially welfare cost estimate, are to the values of this fixed parameter. In reality, even if we as econometricians may assume the knowledge of the structure of the DGP, not being able to precisely calibrate/parameterise additional parameters like the capital share and/or a depreciation rate in this example results in varying degrees of estimation bias. To incorporate the problem into our analysis, we consider two cases. First, we fix $\hat{\delta}_j^i$ at a value in [0.005,0.05] and estimate $\widehat{\Omega}_j^i \equiv \{\hat{A}_j^i, \hat{\alpha}_j^i\}$. We will refer this case as the δ-Parameterised case. By considering a range of values of the deprecation rate, we can investigate the sensitivity of the estimated parameters to the fixed parameter. In the second approach, we fix $\hat{\alpha}_j^i$ at a value in [0.2,0.4] and estimate $\widehat{\Omega}_j^i \equiv \{\hat{A}_j^i, \hat{\delta}_j^i\}$. We will refer this case as the α-Parameterised case. The biases in structural parameter estimates in these two cases are then induced by setting the fixed parameter value higher or lower than the true value as can happen in empirical applications.

After obtaining the estimates of structural parameters, we calculate the estimated welfare cost, denoted by

$1 - \Delta(\widehat{\Omega}_j^i; \tau_*, \tau)$, of each replication j and each experiment i. The bias is defined as the difference between the estimated value and the true value. For example, the bias in the structural parameter estimate for a replication j in an experiment i is $B_j^i = \widehat{\Omega}_j^i - \Omega^i$. For each experiment i, we also compute the mean bias which is defined as $MB^i = {}^1/_N \sum_{j=1}^{N} B_j^i$ and use this statistic in the Response Profiles discussed in the next section.

Response Profile Estimation

We estimate the Response Profiles of the bias in welfare cost estimate to summarise the results of Monte Carlo experiments and examine the relationship between the biases in structural parameter and welfare cost estimates. Details on alternative applications of Response Profiles can be found in Hendry (1984) and Davidson and MacKinnon (1993).

By treating the result of each Monte Carlo experiment as a single observation, we can estimate the true response function by some low order polynomial function, $\Psi(\cdot)$. Each experiment is replicated N times and we use mean bias (MB^i) as the estimator of a bias in each experiment. By including simulation results of all tax policies we consider, we have 4,000 observations. Note that we estimate the Response Profiles separately for the estimation results from the δ-Parameterised case and the α-Parameterised case. The dependent variable in the Response Profile is the mean bias of welfare cost estimates (MB_W) and the explanatory variables are the mean biases of structural parameter estimates: a utility of one unit of leisure (MB_A), a capital share (MB_α) and a depreciation rate of capital (MB_δ). We include up to a third degree of polynomial for all explanatory variables to allow for a flexibility in the Response Profiles function. We also introduce dummy variables $\{d, dp_1, dp_2, dp_3, dp_4\}$ into the regression to capture interesting features of the Response Profiles. The dummy variable d takes the value of 1 if the mean bias of welfare cost estimates is negative and takes value of 0 otherwise. This dummy variable allows us to capture any asymmetric response of the bias in welfare cost estimates to the biases in structural parameter estimates. That is, we found that the bias in structural parameter estimates of the same magnitude with opposite signs result in quantitatively different impacts upon the bias in welfare cost estimate. The remaining dummy variables, $\{dp_1, dp_2, dp_3, dp_4\}$, are assigned to each tax policy to capture the possibility of the change in responses of bias in welfare cost estimate across all tax policies.

Finally, we are tentatively led to the following general specification of the form, $\Psi(MB_A^i, MB_\alpha^i, MB_\delta^i)$. The coefficients corresponding to each explanatory variable in the fitted equation can be estimated by Ordinary Least Squares.

$$\mathrm{MB}_\mathrm{W}^\mathrm{i} = \Psi\left(MB_A^i, MB_\alpha^i, MB_\delta^i\right) = \sum_{j=1}^{4} dp_j \left\{\mu_j + \sum_{i=1}^{3} \theta_{j,i} MB_A^i + \sum_{i=1}^{3} \theta_{j,i+3} MB_\alpha^i + \sum_{i=1}^{3} \theta_{j,i+6} MB_\delta^i + d\left(\mu_{j,0} + \sum_{i=1}^{3} \theta_{j,0i} MB_A^i + \sum_{i=1}^{3} \theta_{j,0i+3} MB_\alpha^i + \sum_{i=1}^{3} \theta_{j,0i+6} MB_\delta^i\right)\right\}$$

where

$d = 1$ if $MB_W^i < 0$

$dp_1 = 1$ if $\Delta_\tau =$ -20%

$dp_2 = 1$ if $\Delta_\tau =$ -10%

$dp_3 = 1$ if $\Delta_\tau =$ +10%

$dp_4 = 1$ if $\Delta_\tau =$ +20%

and equal to zero otherwise.

We use Bayesian Information Criterion (BIC) to select the specification as this measures the relative goodness of fit of the response function while also penalizing over parameterisation. After estimating several different models, we choose the one which minimizes BIC.

Given the estimated Response Profiles, we can obtain useful information on the relationship between the structural parameter and welfare cost estimation biases. The slopes of the estimated Response Profiles help us identify which structural parameter estimation bias induces the most impact to the bias in welfare cost estimate, assuming other biases in structural parameters are constant. Moreover, the gradient of the estimated Response Profiles represents the steepness and direction of the slope. The magnitude of the gradient provides how fast the bias in welfare cost estimate increases in the direction. We can use this representation to identify the portion of parameterised parameter space that induce the least impact to the bias in welfare cost estimate.

Results and Discussion

This section begins with a discussion of the impact of small sample bias on the bias in welfare cost estimates. We then concentrate our analysis to the impact of parameter identification issues. Finally, we make suggestions on how to calibrate and estimate structural parameters given this framework to reduce the bias in welfare cost estimate. Note that we focus our analysis on the experiments of the δ-Parameterised case and do not elaborate the results of the α-Parameterised case as the implications are similar.

Impact from the Small Sample Bias

Table 1 summarises means of the mean bias in welfare cost estimate for each tax policy given data of different sample sizes in the δ-Parameterised Case. The z-values show the evidence that there are significant spill overs from the biases in structural parameter estimates to the bias in welfare cost estimates. With a sample size of 100, we obtain the largest bias in welfare cost estimates for all tax policies. Interestingly, even though we obtain a smaller mean of the mean bias in welfare cost estimate by increasing the sample size from 100 to 200, there is a little difference when we increase the sample size from 200 to 1000. Hence, we can simply say that the impact from the small sample bias to the bias in welfare cost estimate is eliminated when the sample size is at least 200. However, it is evident that the bias in welfare cost estimate is still significantly different from zero. Recall that, together with the small sample bias, our parameter estimates also suffer from the lack of identification. Consistent with Canova and Sala (2009), the bias present in partially and non-identified parameters would spill to the welfare cost estimate and remain significant even in the large sample. This shows how important the parameter identification issue is to the estimation of the welfare cost.

δ-Parameterised Case				
Sample Size (T)	The mean of MB_W			
	$\Delta\tau =$ -20%	$\Delta\tau =$ -10%	$\Delta\tau =$ +10%	$\Delta\tau =$ +20%
100	0.187% (10.735)	0.091% (14.763)	-0.073% (-25.658)	-0.124% (-34.508)
200	0.128% (7.336)	0.054% (8.643)	-0.030% (10.127)	-0.036% (-9.986)

1000	0.127% (7.263)	0.053% (8.502)	-0.028% (-9.742)	-0.0260% (-9.342)

Notes: The standardized z-value in parentheses is $z = MB \Big/ \sqrt{\frac{MSD}{N*M}}$.

Table 1: The Mean of Mean Biases in Welfare Cost Estimates with Different Sample Sizes

Importance of Parameter Identification Problems

For the nonidentified parameter, the utility of leisure (A), we encounter serious biases in the utility of leisure estimates as the likelihood is noninformative on this parameter. However, the welfare cost estimate does not depend on the value of utility of leisure and hence the biases in utility of leisure do not translate into the bias in welfare cost estimates. The functional form of utility specified in the model therefore plays an important role in determining which structural parameter is important for the estimation of welfare costs. It is possible that, by deviating from this functional form, the biases in utility of leisure estimates might matter and cause a greater impact to the bias in welfare cost estimate than what we have in this particular exercise.

The capital share (α) and the depreciation rate of capital (δ), on the other hand, play an important role in estimating welfare cost as they enter directly into the welfare cost function. Figures 2 and 3 illustrate the Response Profiles of biases in capital share and welfare cost estimates respectively. The surface plots display mean bias of parameter estimates for each experiment where we fix the depreciation rate at the values along the x-axis against the true values along the y-axis. The right (left) area of the diagonal indicates a negative (positive) bias in the depreciation rate estimate as we fix a value of the depreciation rate lower (higher) than a true value. Within the same diagrams, the dot plots display estimation results of each replication and depict the variation of biases in parameter estimates resulting in each experiment. From these figures, we can make the following observations.

As one would expect, if we correctly fix the value of the depreciation rate, we can pin down the estimated capital share correctly with only a small variation. As can be seen from the diagonal of the surface plot in Figure 2, fixing a value of the depreciation rate without bias leads to no bias in the estimated capital share. On the other hand, by observing the off-diagonal area of the surface plot, a nonzero bias in the depreciation rate, in turn, induces a nonzero bias in the estimated capital share. In particular, a negative (positive) bias in the depreciation rate gives a negative (positive) bias in capital share estimate. Hence, the Response Profile suggests that there is a positive relationship between the capital share and depreciation rate of capital estimation bias.

Due to the problem of identification in the key structural parameters, the Response Profiles of bias in welfare cost estimates in Figure 3 illustrate how the biases in these structural parameter estimates translate into the bias in welfare cost estimate across all tax policies. Similar to Figure 2, no bias in structural parameter estimates leads to no bias in welfare cost estimates. Once we induce biases in structural parameter estimates, the Response Profiles of bias in welfare cost estimates suggest the following. When the income tax rate declines below the baseline tax rate; $\Delta\tau$ = -10% and $\Delta\tau$ = -20%, there is a negative nonlinear and asymmetric relationship between the structural parameter and welfare cost estimation bias. This pattern is displayed by top panels in Figure 3. On the other hand, we have a positive nonlinear and asymmetric impact of the biases in structural parameter estimates to the bias in welfare cost estimates when the income tax rate rises above the baseline tax rate; $\Delta\tau$ = +10% and $\Delta\tau$ = +20%. This pattern is displayed by bottom panels in Figure 3. Here the asymmetric relationship means that, for instance, a negative bias in the depreciation rate estimate of 0.04 only produces an upward bias in the welfare cost estimate of 2% whereas having a positive bias in the depreciation rate estimate of the same size pulls the bias in welfare cost estimate as low as -3% when the income tax rate decreases by 10%.

Another interesting feature we can observe from the Response Profiles of bias in welfare cost estimate is the change in the slopes of bias in welfare cost estimates to the biases in structural parameter estimates across tax policies. Table 2 summarises the Monte Carlo experiments for each tax policy as the range of mean biases of welfare cost estimates. We can see that the ranges become wider as the income tax rate deviates further away from the baseline tax policy. Recall Equation (8), the multiplicative terms between a new tax rate and estimated structural parameters amplify the impact of the biases in structural parameter estimates to the bias in welfare cost estimate. Consequently, even with only small biases in structural parameter estimates caused by the lack of identification, they can reduce the accuracy of welfare estimates.

In sum, the problem of parameter identification is not only crucial for the estimation of the structural parameters but also for the estimation of welfare cost as this is a function of these structural parameter estimates. Consequently, the lack of identification of key structural parameters specified in the model can bias the welfare cost estimates and eventually mislead policy implications.

Implications from Response Profile Estimation

Recall we included several dummy variables into the Response Profile functions and estimate the coefficients using Ordinary Least Squares. Given the estimated Response Profile functions of both cases, we can make the following implications.

In the δ-Parameterised case, tests on the coefficients suggest that the slopes of the bias in welfare cost estimate to biases in structural parameter estimates change across all tax policies and there are asymmetric responses of the bias in welfare cost estimate. Figures 4 and 5 plot the slopes and the gradients of the Response Profiles respectively. We can see that, in this particular framework, the bias in welfare cost estimates is most sensitive to the bias in the depreciation rate of capital estimate.

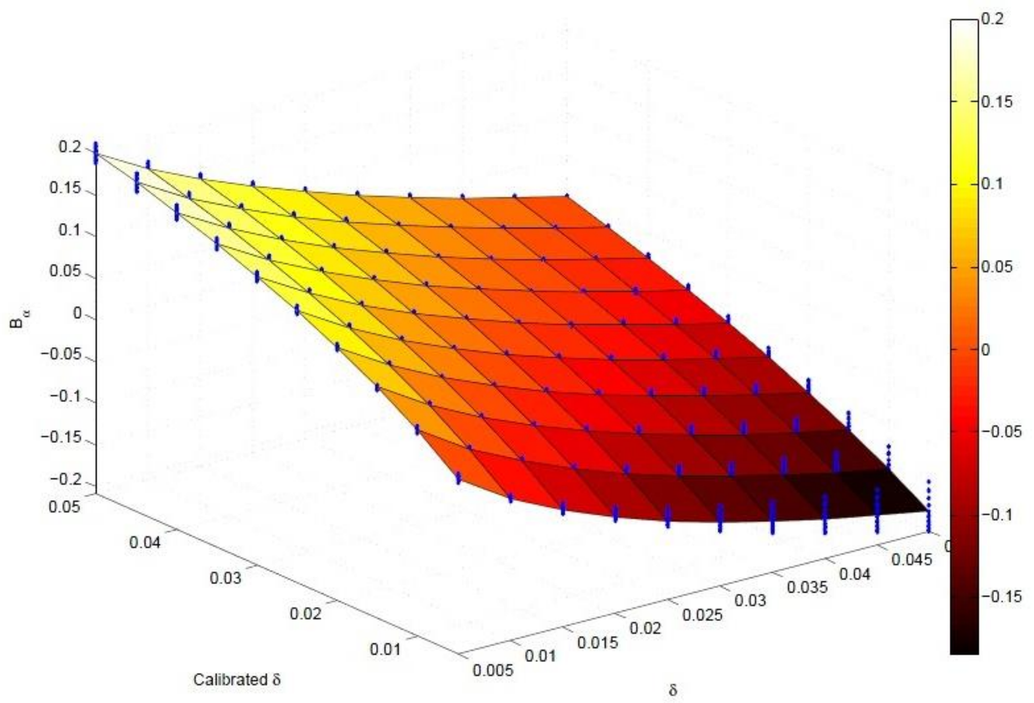


Fig. 2: Response Profile of Biases in Capital Share (B_α) to Biases in Depreciation Rate (B_δ)

δ-Parameterised Case	
Tax Policies	The Range of Mean Bias of Welfare Cost Estimates
$\Delta\tau$ = -10%	-3.077% - 1.940%
$\Delta\tau$ = -20%	-8.784% - 5.394%
$\Delta\tau$ = +10%	-0.903% - 1.464%
$\Delta\tau$ = +20%	-1.163% - 1.919%

Note: The baseline fiscal policy is $\tau_* = 30\%$

Table 2: The Range of Mean Biases in Welfare Cost Estimates

The slopes also display a greater impact of biases in structural parameter estimates on bias in welfare cost estimates when the tax rate deviates further away from the baseline tax policy. Furthermore, as we investigate the sensitivity of the parameterised values of depreciation rate in this case, the magnitudes of the gradients of the Response Profiles show that the bias in welfare cost estimates becomes less sensitive to biases in structural parameter estimates in the area where we fix values of the depreciation rate lower than the true values; shown as the southeast area of the Response Profiles in Figure 5.

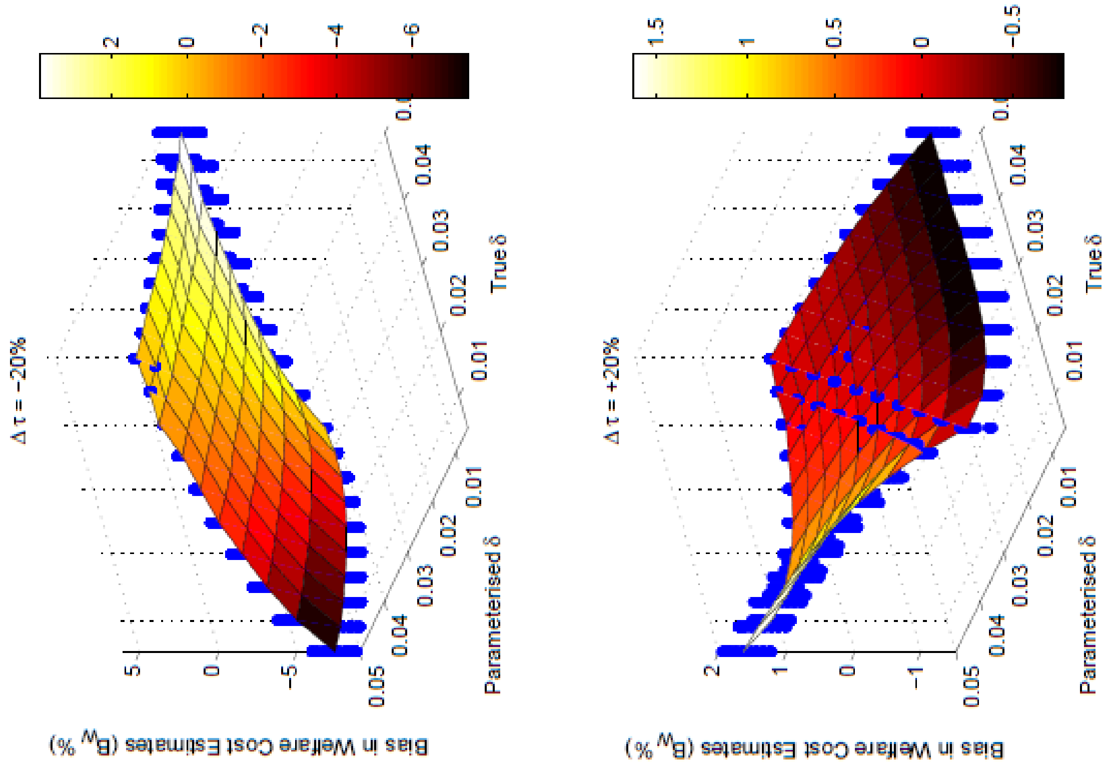
We repeat the same exercises for the α-Parameterised case in which we fix a parameter value of the capital share (α) and obtain the estimated Response Profiles. The experiment also suggests that the bias in depreciation rate estimates induce the most significant bias in welfare cost estimate and the impact of biases in structural parameter estimates becomes larger when the tax rate deviates away from the baseline tax policy. However, in this case, the bias in welfare cost estimate become less sensitive to biases in structural parameter estimates when fixing values of the capital share higher than true values.

The estimated Response Profiles for both cases then suggest that fixing the depreciation rate of capital too high, or the capital share too low, leads to a larger bias in welfare estimates. We can therefore exploit this additional information regarding the relationship by imposing priors on these two parameters to improve the estimation of welfare cost estimate.

Conclusion

Welfare cost estimates depend on the structural parameter estimates specified in a

DSGE model. This paper studies the implications for welfare analysis of structural parameter estimate bias. We implement Monte Carlo experiments and estimate Response Profiles to approximate the size of biases and study the relationship between structural parameter and welfare cost estimation biases. The findings can be summed up as follows. The relationship is not linear and the bias in welfare cost estimates respond very differently to the bias of different structural parameter estimates. The problem of parameter identification is important as it can distort the parameter estimates even in a large sample size. Moreover, the nonlinearity of the welfare cost function amplifies the impact of parameter estimation bias on welfare cost estimation bias and reduces the accuracy of policy experiments. In this particular exercise, the estimated Response Profiles suggest that we can reduce the bias in welfare cost estimates by setting the depreciation rate of capital too low or setting the capital share too high.



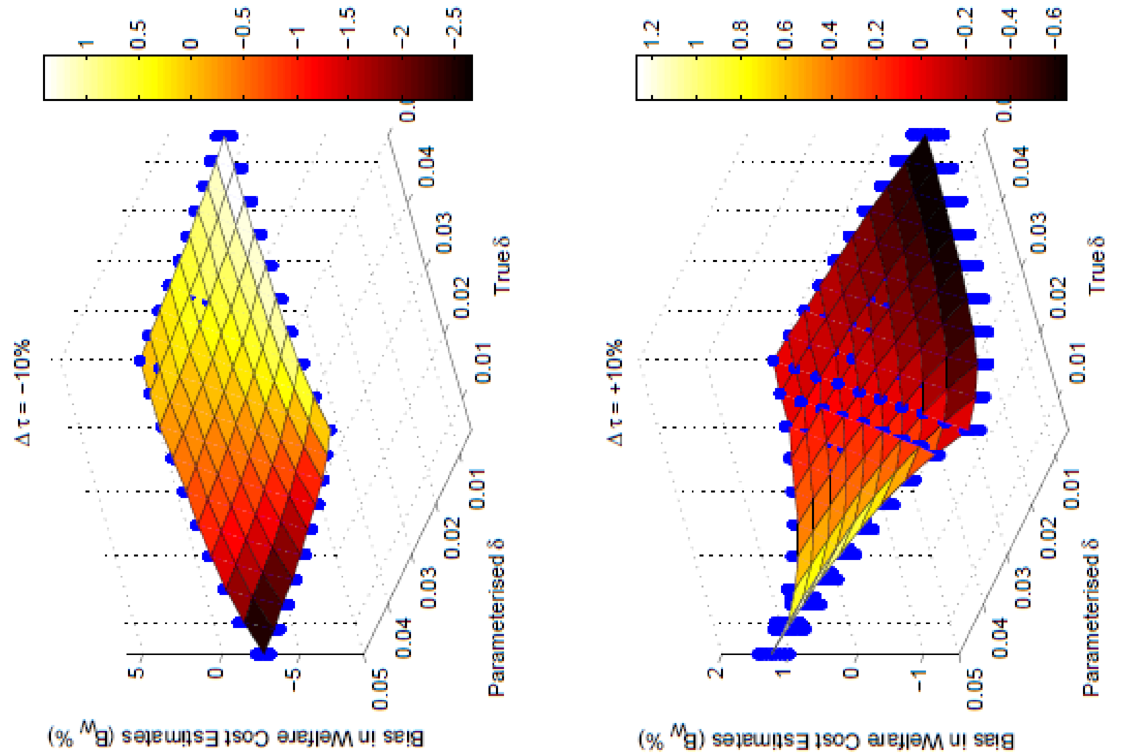


Fig. 3: Response Profiles of Biases in Welfare cost ($\boldsymbol{B_W}$) to Biases in Structural Parameter Estimates for each Tax Policy in the $\boldsymbol{\delta}$–Parameterised Case

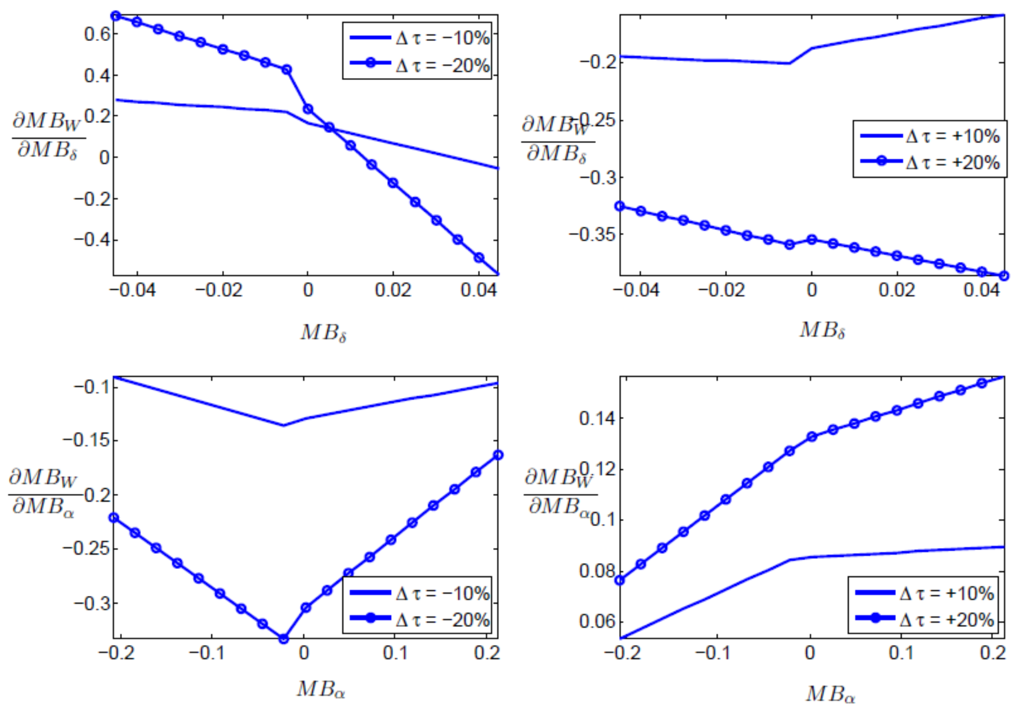


Fig. 4: The Slopes of Response Profiles with respect to each Structural Parameter

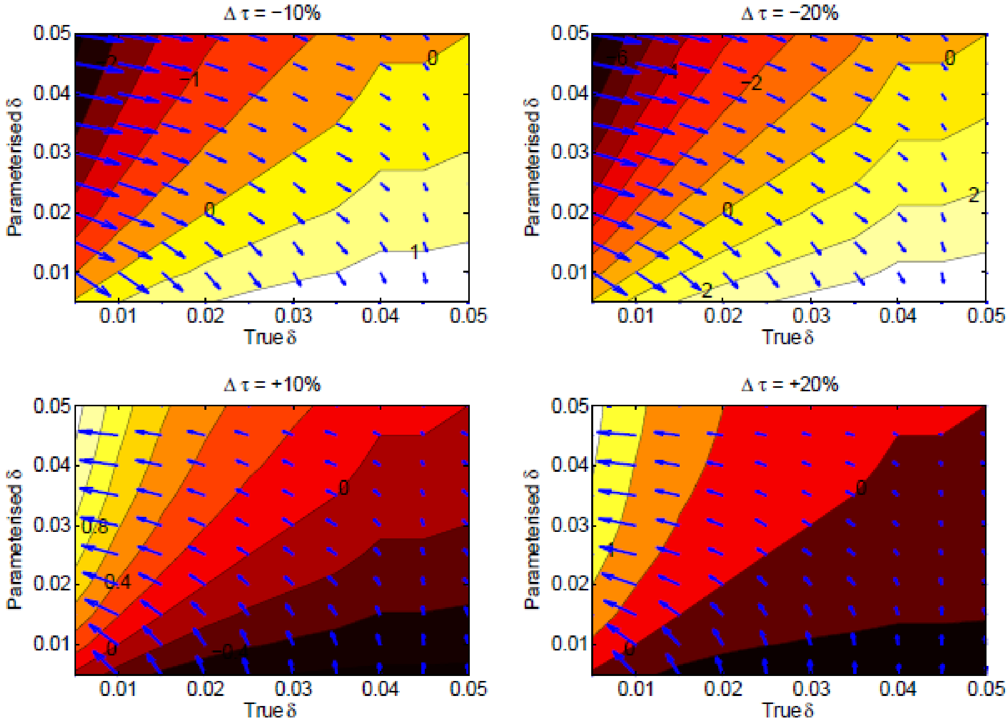


Fig. 5: The Gradient of Response Profiles for each Tax Policy in the δ–Parameterised Case

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