ENHANCING STUDENTS' MATHEMATICAL ACHIEVEMENT BY APPLYING CRITICAL THINKING IN MATHEMATICS AND COOPERATIVE LEARNING

การส่งเสริมผลสัมฤทธิ์ทางคณิตศาสตร์ของนักเรียน โดยการประยุกต์ใช้การคิดอย่างมีวิจารณญาณทางคณิตศาสตร์และการเรียนรู้แบบร่วมมือ

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Pongsakorn Plubsiri[1*] Supotch Chaiyasang[2]
พงศกร พลับศิริ[1*] สุพจน์ ไชยสังข์[2]
*Corresponding Author, E-mail: pongsakorn.bom@gmail.com

Abstract

The objectives of this classroom action research were to enhance students' mathematical achievement and to survey students' satisfaction in learning by applying critical thinking in mathematics and cooperative learning. The participants were 50 grade 10 students who enrolled in the first semester of the academic year 2016 at a secondary school in Bangkok, Thailand. The topic used in this study was Elementary of Sets. The instruments were 8 lesson plans integrated critical thinking in mathematics and cooperative learning. Teaching and learning lasted 12 periods with 50 minutes in each period. There were three cycles of action plan. Data were collected from mathematical achievement test, satisfaction survey, and teacher's reflections. Data were analyzed by mean, percentage, mode, and standard deviation. The results showed that 88 percent of all students passed achievement test which satisfied with the hypothesis set by the researcher. That is at least 70 percent of all students pass achievement test developed by the researcher. From students' satisfaction survey, it showed that students had high levels of satisfaction (very satisfied or satisfied level) in self-efficacy category, in students' participation category, and in students' understanding category.

Keywords: Critical Thinking in Mathematics, Cooperative Learning

[1]Master Degree, student Mathemation Program, International College, Sunandha Rajabhat University
[2]Asst.Prof.Dr., International College, Sunandha Rajabhat University

บทคัดย่อ

วัตถุประสงค์ของการวิจัยปฏิบัติการในชั้นเรียนนี้เพื่อเพิ่มผลสัมฤทธิ์ทางการเรียนคณิตศาสตร์ของนักเรียน และสำรวจความพึงพอใจของนักเรียนในการเรียนด้วยการประยุกต์ใช้การคิดอย่างมีวิจารณญาณในวิชาคณิตศาสตร์ และการเรียนแบบร่วมมือ กลุ่มนักเรียนที่ใช้ในการวิจัยครั้งนี้เป็นนักเรียนระดับชั้นมัธยมศึกษาปีที่ 4 ของโรงเรียนระดับมัธยมศึกษาแห่งหนึ่ง ในกรุงเทพมหานคร ในภาคเรียนที่ 1 ปีการศึกษา 2559 จำนวน 50 คน ในหัวข้อเรื่องเซตผู้วิจัยพัฒนาแผนการสอน 8 แผน ที่บูรณาการการคิดอย่างมีวิจารณญาณในวิชาคณิตศาสตร์ และการเรียนแบบร่วมมือเพื่อเพิ่มผลสัมฤทธิ์ทางคณิตศาสตร์และสำรวจความพึงพอใจของนักเรียน การเรียนการสอนใช้เวลา 12 คาบ คาบละ 50 นาที มีการสะท้อนผลการสอน 3 ครั้ง ข้อมูลรวบรวมจากแบบทดสอบวัดผลสัมฤทธิ์ทางการเรียนคณิตศาสตร์ของนักเรียน แบบสำรวจความพึงพอใจและการสะท้อนผล วิธีการวิเคราะห์ข้อมูลใช้ ค่าเฉลี่ย ร้อยละ ฐานนิยม และส่วนเบี่ยงเบนมาตรฐาน ผลการวิจัยพบว่า นักเรียนร้อยละ 88 ของนักเรียนทั้งหมดมีผลสัมฤทธิ์ทางการเรียนคณิตศาสตร์ผ่านเกณฑ์ร้อยละ 70 ของคะแนนทั้งหมด และจากการสำรวจความพึงพอใจของนักเรียนพบว่า นักเรียนมีความพึงพอใจในระดับสูง (ระดับความพึงพอใจมาก หรือพึงพอใจ) ในด้านสมรรถภาพส่วนตัวนักเรียน ในด้านการร่วมมือกันของนักเรียนและในด้านความเข้าใจของนักเรียน

คำสำคัญ: การคิดอย่างมีวิจารณญาณในวิชาคณิตศาสตร์ การเรียนแบบร่วมมือ

Introduction

According to the Basic Education Core Curriculum B.E. 2551 (A.D.2008), mathematics is one learning area required for Grade 1 to Grade 12 students to possess bodies of knowledge, learning processes and skills and desirable characteristics as indicated in learning standards. Learning processes and skills are problem solving, reasoning and proof, communication, connection, and representation (Ministry of Education, 2008). However, students' mathematical achievement from various tests were lower than that of the expectation of the public. For example, PISA (Programme for International Students Assessment) in 2012 revealed that the average mathematics score of Thai students was below than OECD (Organisation for Economic Co-operation and Development) average (OECD, 2014). In 2015, TIMSS (Trends in International Mathematics and Science Study) exposed that mathematics average score of Thai students was categorized in the low international benchmark (The Institute for the Promotion of Teaching Science and Technology [IPST], 2016). O-NET (Ordinary National Education Test) disclosed that the average score of mathematics was not satisfactory. The averages of the mathematics scores on grade 12 were 21.74%, 26.59%, and 24.88% in 2014, 2015, and 2016 respectively (The National Institute of Education Testing Service, 2016).

In a pre-practicum course, the researcher observed mathematics classes and talked to mathematics teachers. They mentioned that students had problems in solving problems. Students were able to find solutions for problems similar to given examples, but they could not solve

problems if the teacher gave more complex problems. Students' mathematical achievement did not reach the expected level. In talking with students, most of them felt they could not connect contents studied in class. Some contents were difficult for them to understand. Before the beginning of the practicum course, the researcher talked with teachers to know about some problems in teaching mathematics. The researcher found that the problems were about low achievement, understanding, reasoning, and problem solving.

From engineering students' problems in understanding mathematics contents, solving engineering mathematics, and student's low achievement, Jacob (2012) in his article, Mathematical Achievement and Critical Thinking Skills in Asynchronous Discussion Forums of Engineering Mathematic, suggested that critical thinking in mathematics would be a way to help students understand and solve mathematics problems. It could help students to think and to solve mathematics problems and increase mathematical achievement. In the 1999 yearbook of National Council of Teachers of Mathematics (NCTM), Krulik and Rudnick, Developing Mathematical Reasoning in Grade K–12, stated critical thinking as the following:

Critical thinking is thinking that examines, relates, and evaluates all aspects of the situation or problem. It includes gathering, organizing, remembering, and analyzing information. Critical thinking includes the ability to read with understanding and to identify extraneous and necessary material. It also means being able to draw proper conclusion from a given set of data and being able to determine inconsistencies and contradictions in a set of data. Critical thinking is analytical and reflexive. (Krulik and Rudnik, 1999, p. 139)

From the above quotation, there are some key components of critical thinking in mathematics such as organizing and analyzing information, understanding and identifying necessary material, drawing proper conclusion, determining contradiction, and analysis.

Monteleone, White & Geiger (2018) in the proceeding of the 41st annual conference of the Mathematics Education Research Group of Australasia also stated the components of critical thinking in mathematics as follows:

1. Use the knowledge of mathematics and mathematical processes
2. Use mathematical and other understandings to generate, and create new ideas
3. Identify and performs many ways to solve mathematical problems
4. Provide reasons or judgments
5. Use mathematical strategies to prove the answer is possible
6. Self-evaluate, using mathematical evidence and reasoning
7. Build on ideas through explanation, questioning, and organizing.

For practical use in teaching and increasing students understanding and achievement, the researcher identified components of critical thinking in mathematics as follows: 1) building mathematical ideas through questioning, exploring, explaining, and organizing, 2) using mathematical understandings to generate and create new ideas, 3) providing reasons or judgments, and 4) performing various ways to solve mathematical problems.

To develop students' critical thinking in mathematics, the researcher taught concepts and basic knowledge to students. After that, the researcher developed critical thinking by using questions. The questions used were in four types: 1) question that investigates understanding, 2) questions for explaining, reasoning, or argumentation, 3) questions for student to proof or disproof a mathematical statement, 4) questions for analyzing, solving problems, or exploring. For example, in set operation properties, the researcher asked students to show that $(A \cup B)' = A' \cap B'$. Students might use examples, reasoning, argumentation, or proof to show that both sides are equal. Another example, the researcher asked students to find or to generate new properties of set operations. They used their knowledge, ideas, and techniques (using Venn diagrams) to generate new properties. These are the ways that critical thinking in mathematics could improve students' mathematical achievement. The researcher also wanted students to discuss their reasoning, argumentation, and proof. Therefore, the researcher decided to use cooperative learning because it helped students discuss their idea with their classmates.

Cooperative learning is a learning situation in which two or more students work together to complete a common task (Siegel, 2005). It helped students exchange their idea with classmates and find ways to solve problems. There are many strategies in cooperative learning such as the jigsaw, student teams achievement divisions (STAD), teams-games-tournaments (TGT), and think-pair-share. Because of the limited class space and a large number of students (50 students), the researcher chose the method that students could work in small groups. Think-Pair-Share would be the appropriate strategy to be used in this study. Think-Pair-Share strategy developed by Frank Lyman the professor of the University of Maryland in 1981 (Slone and Mitchell, 2014). Using critical thinking in mathematics and think-pair-share might enhance students' mathematical achievement because they could enhance students' cognitive behaviors, understanding, problem solving, and analyzing.

From the students' problems in learning mathematics and students' low achievement, the researcher aimed to apply critical thinking in mathematics and cooperative learning (think-pair-share) to enhance students' mathematical achievement.

Research Objectives

1. To enhance students' mathematical achievement by applying critical thinking in mathematics and cooperative learning.

2. To survey students' satisfaction in learning by applying critical thinking in mathematics and cooperative learning.

Hypothesis: After applying critical thinking in mathematics and cooperative learning, at least 70% of all students passed achievement test developed by the researcher. The criterion for each student to pass achievement test was to gain at least 70% of total score.

Conceptual Framework

Action research is conducted by the researcher to solve specific problem(s) in a specified area or by practitioners for improving their practices. Classroom action research is also conducted by teachers as researchers to solve problem(s) in classroom or to improve their own teaching. It sometimes creates ideas for grounded theory which are bases for main theory. Actually, in action research, the researcher neither identifies sample nor population because the researcher aims to solve problems in a specific area and also does not aim to generalize the research results. In teaching mathematics, the problems fixed with the classes are about students' understanding, reasoning, making judgment, proving or disproving, and problem solving. From various studies and ideas from experts, this classroom action research attempted to identify components of critical thinking in mathematics and to integrate those components in lesson plans for enhancing students' achievement. The researcher adopted the action research model presented by Kemmis and McTaggart (Koshy, 2005) which involves spiral and recursive steps of: 1) planning a change, 2) acting and observing the processes, 3) reflecting on these processes, and 4) revised the plan.

Critical thinking in mathematics and cooperative learning help students in understanding, explaining, reasoning, argumentation, exploring, analyzing, proof or disproof a mathematical statement, and solving problems. These abilities can enhance students' mathematical achievement.

Research Methodology

This research took model of classroom action research with three cycles to enhance students' mathematical achievement and to survey students' satisfaction in learning by applying critical thinking in mathematics and cooperative learning. The participants in this study were 50 grade 10 students in the first semester of academic year 2016 at a secondary school in Bangkok, Thailand.

Variables

Independent variable was applying critical thinking in mathematics and cooperative learning.

Dependent variables were students' mathematical achievement and students' satisfaction.

Research Instruments

The instruments used in this study were consisted of lesson plans, achievement test, satisfaction survey, and teacher's reflections.

1. Lesson plans: There were 8 lesson plans on Elementary of Sets applied critical thinking in mathematics and cooperative learning. The contents were 1) introduction of sets, 2) types of sets, 3) equal sets and equivalent sets, 4) subsets, power sets, and relative universe 5) Venn diagrams, 6) union and intersection, 7) complement and difference, 8) set operation properties, and 9) solving problems by using knowledge about Sets. At the end of periods 2, 5, and 7, the researcher reflected teaching and students' behaviors for improving in the next periods.

2. Achievement test: Mathematical achievement test was developed by the researcher. It was composed of 10 multiple-choice items for 10 points, and 3 written tests for 10 points. This test covered all the topics used in this study. The Index of Item Objective Congruence (IOC) was used to measure the congruence between learning objectives and the test items. The IOC of this test was measured by 3 experts. For this achievement test, the value of IOC in multiple-choice items was 0.97, but IOC in written tests was 0.89.

From 6 cognitive domains presented by Anderson and Krathwohl who presented the revised Bloom's taxonomy, the researcher considered testing students only 4 cognitive domains (remember, understand, application, and analysis) as presented by Anderson and Krathwohl (Anderson & Krathwohl, 2001).

3. Satisfaction survey: This rating-scale survey was a 5-point Likert scale. It was a teacher-made survey adapted from Ali, Ali, & Farag (2014), Noohi, Komsari, Nakhaee, & Yazdi (2013), and Lester (2007) to survey student satisfaction in learning by applying critical thinking in mathematics and cooperative learning. There were 10 items that can be categorized into 3 groups: self-efficacy, students' participation, and students' understanding. The survey items were modified according to the suggestions given by the expert teachers. The value of IOC in the satisfaction survey was 0.98. The questions used in this research were shown as follows:

Category 1: Self-efficacy

Item 1 I feel confident in learning set though the learning.

Item 2 I believe that this learning model can improve my mathematics abilities.

Item 3 During the learning, I can work with my partner.

Category 2: Students' participation

Item 4 I feel that this learning model makes me participate more in classroom.

Item 5 I listen carefully to my classmate's opinion.

Item 6 I think that discussion with another student during this learning model assists me to express my ideas frequently in class.

Category 3: Students' understanding

Item 7 I think that exchanging my opinion with other students helps me to acquire better understanding of mathematics concepts.

Item 8 I like this learning model because it enhances mathematics understanding.

Item 9 I got understanding of mathematical concepts from the learning more than usual instruction.

Item 10 I can solve more problems with partner than I could do by myself.

4. Teacher's reflections: There were three reflections to improve teaching and learning activities. In each reflection, the researcher reflected the following aspects: students' problems in classroom, teaching, and improvement plan.

Data Collection

The data were collected and analyzed in the following steps.

1. The data collection of this study was performed during 12 periods of teaching in set topic, as described in the last section. During the intervention period, the participants received learning instruction by applying critical thinking in mathematics and cooperative learning.

2. At the end of lesson plan 2, 5, and 7, the researcher made the reflections on students' problems in classroom and teaching.

3. After finishing all lessons, each participant was asked to take the achievement test and to complete the satisfaction survey.

Data Analysis

Quantitative data: The data from achievement test, and satisfaction survey in each domain were analyzed by using means, percentage, mode, and standard deviation.

Qualitative data: The researcher analyzed qualitative data from researcher's reflection consisted of three aspects: students' problems in classroom, teaching, and improvement plan.

Research Results

Results of students' mathematical achievement

Results are shown in table 1, table 2, and table 3. Table 1 is the result of students' scores in achievement test. The next tables are the result of student's scores in each behavior in multiple choices part and written test part. They were shown in table 2, and table 3 respectively.

Table 1 Results of mathematical achievement test

No. of students (n)	Total score	Mean	S.D.	No. of passed students	Percentage of passed students
50	20	14.96	1.43	44	88.00 %

Table 1 showed mean of students' scores, standard deviation of total score, the number of students who passed, and the percentage of passed students which were 14.96, 1.43, 44, and 88.00 respectively. 44 students or 88% of all students passed the criteria set by the researcher. That was more than 70% (88%) of students acquired at least 70% of total score. Therefore, critical thinking in mathematics and cooperative learning could enhance students' mathematical achievement which satisfied with research hypothesis.

Table 2 Students' scores in four cognitive behaviors of achievement test (multiple choices)

Cognitive Behaviors	Number of Items	Total Score(s)	Mean	Percentage of Total Score	S.D.
Remember	1	1	1	100	0
Understand	3	3	2.68	89.33	0.51
Application	2	2	1.58	79	0.57
Analysis	4	4	2.58	64.50	0.85

Table 2 revealed that achievement test scores on four cognitive behaviors: remember, understand, application, and analysis were high (greater than 60%). The highest mean score was remember (100%). The next high cognitive behaviors were understand (89.33%), and application (79%). For the analysis behavior, it was also high (64.50%).

Table 3 Students' score in four cognitive behaviors of achievement test (written tests)

Cognitive Behaviors	Number of Items	Total Score(s)	Mean	Percentage of Total score	S.D.
Remember	0	0	-	-	-
Understand	0	0	-	-	-
Application	2	6	4.48	74.67	1.06
Analysis	1	4	2.64	66	1.32

Table 3 revealed that achievement test scores on four cognitive behaviors: remember, understand, application, and analysis were high (greater than 60%). The highest mean score was application (74.67%). For the analysis behavior, it was also high (66%).

Results of student's satisfaction survey

Table 4 showed the number of students' responses to items in self-efficacy category.

Table 4 Results of satisfaction survey in self-efficacy category

Item	Rating				
	Very Satis.	Satis.	Neither Satis. nor Dissat.	Dissat.	Very Dissat.
1	23	19	8	0	0
2	17	18	15	0	0
3	19	20	11	0	0

From Table 4, mode of rating was very satisfied in item 1, but modes of rating were satisfied in items 2 and 3. So, students were satisfied in self-efficacy category.

Table 5 showed the rating in students' participation category.

Table 5 Results of satisfaction survey in students' participation category

Item	Rating				
	Very Satis.	Satis.	Neither Satis. nor Dissat.	Dissat.	Very Dissat.
4	21	14	15	0	0
5	18	20	12	0	0
6	21	17	12	0	0

The mode of rating in item 5 was satisfied, while the modes of rating in items 4 and 6 were very satisfied. So, students were very satisfied in students' participation category.

Table 6 Results of satisfaction survey in students' understanding category

Item	Rating				
	Very Satis.	Satis.	Neither Satis. nor Dissat.	Dissat.	Very Dissat.
7	20	16	14	0	0
8	22	21	7	0	0
9	23	17	10	0	0
10	16	19	15	0	0

The modes of rating were very satisfied in items 7, 8, and 9 while the mode in item 10 was satisfied. So, students were very satisfied in this category.

Therefore, satisfaction survey revealed that students were satisfied in self-efficacy category, and they were very satisfied in students' participation category, and in students' understanding category.

Result from teacher's reflection: The researcher reflected three key points.

1. Students' problems in classroom: They were 1) some pairs did not talk with his partner, 2) some students did not express their ideas, and 3) some students did not want to present their idea to the class.

2. Teaching: The problems were 1) encouraging students to pay more attention to classroom activities, and 2) integrating or blending critical thinking components in lessons.to make them easy to understand and challenging.

3. Improvement plans: They were 1) for students' problems in classroom 1), the researcher rearranged the pairs if the students wanted to do, 2) for improvement plan in teaching 1), the researcher randomly chose some pairs to present their work to the class and 3) adjusting next lesson plan according to the problems in previous lessons.

Discussions

The students' mathematical achievement was enhanced because of applying critical thinking in mathematics and cooperative learning. When students studied by this teaching, their learning was developed because these teaching support students' thinking, reasoning, communication, and problem solving.

The results of this study were consistent with Jacob (2012) study which disclosed that critical thinking improved students' mathematical achievement. Moreover, in Chukwuyenum (2013) study, supported that critical thinking skill enhanced students' understanding of mathematics concepts because the skill helped in interpreting, analyzing, and evaluating. In cooperative learning, Anaduaka, Sunday, and Olaoye (2018) studied the effect of cooperative learning which discovered that think-pair-share improved students' mathematical achievement compared to conventional method. Furthermore, for some components of critical thinking in mathematics, many studies showed that they could enhance students' mathematical achievement. Wong (2012) mentioned that questioning improved students' mathematical achievement and questioning affected students' mathematics knowledge and ideas. For reasoning, Maiti (2017) revealed that reasoning was correlated to mathematics achievement. Ayal, Kusuma, Sabandar & Dahlan (2016)

also stated that teaching reasoning resulted in higher mathematical achievement than that of conventional teaching. Meanwhile, in Bhat (2014) research, he found that problem solving was a factor of improving students' mathematical achievement because it encouraged students to think, to discover, and to share their ideas with their classmates.

In this study, an important point was developing lesson plans. Integrating or blending critical thinking in mathematics contents on Elementary of Sets and cooperative learning depended on researcher experiences, previous study, and ideas from experts. The researcher used questioning and think-pair-share to stimulate students' thinking as follows.

1. Questions that investigate understanding

The researcher used questions to check students understanding in each topic. These questions asked about idea, definition, and understanding in Elementary of Set. So, questions in this group aim to build students' mathematical ideas, and students must give reasoning for their answers. For instance, 1) Identify the difference between union and intersection (compare and contrast between two definitions), 2) Is it true that {1, 1, 1} = {1}? Why? If it is true, what is the number of elements in the set {5, {1, 2}, 5, 3, 1, 2} (use definition of equality of sets and make a judgement)?

2. Questions for explaining, reasoning, or argumentation

These questions aimed students to explain the reason and to show their thinking process in each step of solving questions. Questions in this group coincided with other types of questions. For example, questions in 1. students gave the reason for the difference or gave reason why it equal or unequal. Another example, is it true that $A - B = A \cap B'$ (reasoning or argumentation)?

3. Questions to proof or disproof a mathematical statement

Questions in this group are about to show their idea and reason to support their thinking. The researcher used these questions to challenge students to justify whether a mathematical statement is true or not. They used reasoning, argumentation, and proof. For instant, is it true that $A - B = B - A$ (disproof)? Find two ways to justify your answer.

4. Questions for analyzing, solving problems, or exploring

Questions in this group aimed students to solve complicated problems. Students used trial and error to find answer or investigated the pattern of the information that they had. For example, the problems might be 1) show that $A \cap B$ is a subset of A (solving problem), 2) Is it true that $A' \cup B' = (A \cap B)'$ (analyzing)? and 3) can you find new operation properties besides those you have study in class (exploring)?

Think-Pair-Share helped students share their ideas with their pairs. Students who were not good at mathematics might get new ideas from their peers. For example, students did not know or cannot solve the problem by themselves. But think-pair-share helped them to exchange their ideas to pair who understand and can solve the question. They helped their pairs to understand more and clearly. Think-Pair-Share improved students' understanding and enhance their mathematical achievement.

Using the above questions and blending them with think-pair-share could enhance students' critical thinking in mathematics and at the same time enhanced students' mathematical achievement as presented in table 1.

In this study, the researcher specified problems before teaching in the class, and the researcher wanted to solve the problems about students' low achievement and problem solving. Therefore, classroom action research is a method that the researcher used to solve them. Kemmis and McTaggart's model used in this study were spiral and recursive. So, the researcher adapted the plan at the end of each cycle to solve problems found in classroom for the next cycle.

Recommendations

Recommendations for teachers

1. For improving practices, teacher should practice integrating or blending critical thinking in mathematics to other content such as Functions, Calculus, and Statistics.

2. In using think-pair-share strategy to teach in classroom, teacher should be careful about matching students in the following points:

2.1 Students have different traits. Some of them are shy and lack of confidence to talk with their classmates, so teacher should make students feel relax or pair them with friendly students to encourage them to participate in the class.

2.2 Some students are weak at mathematics. Teacher should avoid matching weak students together. That will make students ignore to learn. So, teacher should separate them out and pair with better students to help them learn and exchange their thinking.

Recommendations for future research

1. Critical thinking components in mathematics as presented in this study are complex. Different teachers might use a different blending of critical thinking. The ways of blending will be reflected in lesson plans and such blending will make a classroom lively, exciting, and inspiring. The researcher encourages teachers to develop research lessons applying critical thinking in mathematics in different contents and levels.

2. In using cooperative learning, researcher should study each strategy critically and could identify which strategy is suitable to research aims and classroom environment.

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