การพัฒนากระบวนการเรียนการสอนโดยใช้กระบวนการวางนัยทั่วไป เพื่อส่งเสริมความสามารถในการให้เหตุผลทางพีชคณิตและ การสื่อสารทางคณิตศาสตร์ของนักเรียนมัธยมศึกษาปีที่ 3

Development of Instructional Process by Using the Process of Generalization to Enhance Algerbraic Reasoning Ability And Mathematical Communication of Ninth Grade Students

พรรณทิพา พรหมรักษ์*

บทคัดย่อ

การวิจัยครั้งนี้มีวัตถุประสงค์เพื่อ 1) พัฒนากระบวนการเรียนการสอนโดยใช้กระบวนการวางนัยทั่วไป เพื่อส่งเสริมความสามารถในการให้เหตุผลทางพีชคณิตและการสื่อสารทางคณิตศาสตร์ของนักเรียนมัธยมศึกษาปีที่ 3 และ 2) ศึกษาคุณภาพของกระบวนการเรียนการสอนที่พัฒนาขึ้น โดยพิจารณาจากความสามารถในการให้เหตุผลทางพีชคณิตและความสามารถในการสื่อสารทางคณิตศาสตร์ ดำเนินการพัฒนากระบวนการเรียนการสอนโดยวิเคราะห์และสังเคราะห์ข้อมูลพื้นฐานเกี่ยวกับสภาพปัญหาการเรียนการสอนคณิตศาสตร์ในระดับการศึกษาขั้นพื้นฐาน แนวคิดและทฤษฎีที่เกี่ยวข้อง นำข้อมูลที่ได้จากการศึกษามาสร้างกระบวนการเรียนการสอน แล้วนำไปทดลองใช้กับกลุ่มตัวอย่าง ซึ่งเป็นนักเรียนชั้นมัธยมศึกษาปีที่ 3 โรงเรียนสาธิต “พิบูลบำเพ็ญ” มหาวิทยาลัยบูรพา จำนวน 2 ห้องเรียน จำนวนนักเรียน 79 คน แบ่งเป็นกลุ่มทดลอง 40 คนและกลุ่มควบคุม 39 คน ระยะเวลาในการทดลอง 12 สัปดาห์ เนื้อหาที่ใช้ในการวิจัย คือ ระบบสมการและเศษส่วนของพหุนาม เครื่องมือที่ใช้ในการทดลองคือ แบบวัดความสามารถในการให้เหตุผลทางพีชคณิต และแบบวัดความสามารถในการสื่อสารทางคณิตศาสตร์ วิเคราะห์ข้อมูลโดยใช้ค่าเฉลี่ยเลขคณิต ค่าเบี่ยงเบนมาตรฐาน และค่าสถิติที ผลการวิจัยสรุปได้ดังนี้

1. กระบวนการเรียนการสอนที่พัฒนาขึ้นประกอบด้วยขั้นตอน 4 ขั้นตอน ได้แก่ 1) ขั้นการสร้างความสัมพันธ์ 2) ขั้นการปฏิบัติกิจกรรม 3) ขั้นการสร้างข้อสรุป 4) ขั้นการประยุกต์ความรู้

2. ผลของการทดลองใช้กระบวนการเรียนการสอนที่พัฒนาขึ้น มีดังนี้

2.1 ความสามารถในการให้เหตุผลทางพีชคณิตและการสื่อสารทางคณิตศาสตร์ของนักเรียนหลังเรียนด้วยกระบวนการเรียนการสอนที่พัฒนาขึ้นสูงกว่าก่อนเรียนอย่างมีนัยสำคัญทางสถิติที่ระดับ .05

2.2 ความสามารถในการให้เหตุผลทางพีชคณิตและการสื่อสารทางคณิตศาสตร์หลังเรียนของนักเรียนกลุ่มทดลองสูงกว่ากลุ่มควบคุมอย่างมีนัยสำคัญทางสถิติที่ระดับ .05

* นิสิตระดับดุษฎีบัณฑิต สาขาวิชาหลักสูตรและการสอน คณะครุศาสตร์ จุฬาลงกรณ์มหาวิทยาลัย

2.3 ความสามารถในการให้เหตุผลทางพีชคณิตและการสื่อสารทางคณิตศาสตร์ของนักเรียนกลุ่มทดลองที่เรียนด้วยกระบวนการเรียนการสอนที่พัฒนาขึ้น มีพัฒนาการไปในทางที่ดีขึ้น นักเรียนสามารถสร้างข้อสรุปอย่างเป็นเหตุเป็นผล และสามารถอธิบายแนวความคิดโดยใช้ภาษาและสัญลักษณ์ทางคณิตศาสตร์ได้เป็นอย่างดี

คำสำคัญ : กระบวนการวางนัยทั่วไป, การให้เหตุผลทางพีชคณิต, การสื่อสารทางคณิตศาสตร์

Abstract

The purposes of research were to : 1) develop instructional process by using process of generalization to enhance algebraic reasoning ability and mathematical communications of ninth grade students, and 2) study the quality of the developed instructional process on algebraic reasoning ability and mathematical communications. The researcher developed the instructional process by analyzing and synthesizing fundamental information concerning the state of problems in mathematical instruction at the basic education level. The instructional process was developed based on process of generalization. The developed instructional process was verified by experts and tryout. This study was a quasi-experimental research with two groups pretest-posttest design. The samples of this study were 79 ninth grade students in Piboonbumpen Demonstration School, Burapha University. They were divided into two groups with 40 students in the experimental group and 39 students in the control group. The duration of the experiment was 12 weeks long. The system of equations and fractional polynumials were used in this study. The research instruments were tests of algebraic reasoning ability and mathematical communications. Data were analyzed by using arithmetic mean, standard deviation, and t-test. The findings were as follows:

1. The developed instructional process consisted of 4 steps, namely: 1) relating prior knowledge to new knowledge 2) doing learning activity 3) making conclusion, and 4) applying knowledge.

2. The results of implementing the developed instructional process were:

2.1 algebraic reasoning and mathematical communications abilities of students in the experimental group after learning with developed instructional process were significantly higher than before learning with developed instructional process at .05 level of significance.

2.2 algebraic reasoning and mathematical communications abilities of students in the experimental group after learning with developed instructional process were significantly higher than those of students in the control group at .05 level of significance.

2.3 algebraic reasoning and mathematical communications abilities of students in the experimental group were mathematical developed in positive direction. They could be able to draw mathematical conclusions reasonably, and could elaborate ideas by using mathematical language and symbols effectively.

Keywords: Process of generalization; Algebraic reasoning; Mathematical communication

Introduction

The Basic Education Curriculum B.E.2544 (A.D. 2001) aims to develop learners in 4 areas: morality, intellectual growth, quality of life, and competitive ability. Education management emphasizes on knowledge, thought, capability, morality, learning processes, and social responsibility. The aims are to foster the well balanced development of individual as a learner. One of the focuses of this curriculum is the skills and learning processes in mathematics. Although mathematics is very important for all students, Thai students have low competency in mathematics. The Organization for Economics Co-operation and development studies education system efficiency of member countries. The finding shows that the average of Thai students score is 433 compared to that of the whole project which is 500. The rank of Thailand is 32th from 41 members. Learning Achievement at Elementary and Secondary in Mathematics of Academic Year 2008, that shows percentage of average scores of ninth grade was 32.64. Further more; Thai ninth grade students got 14.52 from the total of 30 in Thailand mathematics national 2008 test. The result of these assessments shows the problems in Thai Education. In mathematics classrooms, pedagogical choices impact student learning. Traditional instruction methods, which include lectures, note taking and memorization are the primary mode of instruction used in schools. Traditionally, the teacher is at the center of the learning process determining what the students must learn and how to learn. The teachers emphasized on students passively received content rather than construct their knowledge. The effectiveness of traditional teaching in this way, the students lacked of skills in mathematics such as thinking, decision making, and communicating.

As this problem caused Thai students having low competency in mathematics, the researcher is, therefore, interested in developing instructional process by using process of generalization to enhance algebraic reasoning ability and mathematical communications of ninth grade students. The research questions are: 1) do the students who learned through the instructional process using process of generalization have higher algebraic reasoning ability and mathematical communications than those who learned through the regular instructional process? 2) do the students who learned through the instructional process using process of generalization have more developments of algebraic reasoning ability and mathematical communications after the experimentation?

Conceptual Frameworks

Generalization

Generalization represents a key element of mathematics and a guiding goal in the mathematics classroom. The process of mathematical generalization involves students in looking across particular cases for meaningful commonalities, such as patterns and structures,and identifying and exposing these relationships (Kaput, 1999, Mason 1996). Mason (1996: 95) described generality as "seeing a generality through a particular and seeing the particular in the general". Kaput (1999) defined generalization as engaging in at least one of three activity : 1) identifying commonality across cases, 2) extending one's reasoning beyond the range in which it originated, or 3) deriving broader results from particular cases.

Ellis (2007) described generalization taxonomy as a method that students create when reasoning algebraic. The descriptions are as below.

Table 1 The generalizing actions of the generalization taxonomy

Type	Description
1. Relating	Relating situation: The formation of an association between two or more problems or situations
	Relating objects: The formation of an association of similarity between two or more present objects
2. Searching	Same relationship: Performing a repeated action in order to detect a stable relationship between two or more objects
	Same procedure: Repeatedly performing a procedure in order to test whether it remains valid for all cases
	Same pattern: Checking whether a detected pattern remains stable across all cases
	Same solution or result: Performing a repeated action in order to determine if the outcome of the action is identical every time
3. Extending	Expanding the range of applicability: Applying a phenomenon to larger range of cases than that from which it originated
	Removing particulars: Removing some contextual details in order to develop a global case.
	Operating: Mathematically operating upon an object in order to generate new cases
	Continuing: Repeating an existing pattern in order to generate new cases
4. Identification or statement	Continuing phenomenon: Identification of a dynamic property extending beyond a specific instance
	Sameness: A statement of commonality or similarity
	General principle: A statement of a general phenomenon
5. Definition	Class of objects: Definition of a class of objects all satisfying a given relationship, pattern, or other phenomenon
6. Influence	Prior idea or strategy: Implementation of a previously developed generalization
	Modified idea or strategy: Adaptation of an existing generalization to apply to a new problem or situation

There are three factors that enhance the effectiveness of this process of generalization, namely: 1) justification, 2) strategy, and 3) constructivism.

Justification

Justification is considered essential component of generalizing activity, as noted by Lannin (2005: 235), generalization cannot be separated from justification. Justification plays two key roles related to understanding student thinking : 1) it allows the teacher to have an insight why a student used a particular strategy, and 2) it provides a window for ascertaining the degree to which students view the generality of their rules (Lins, 2001).

Justification framework that Simon and Blume (1996) developed, drawing from the work of Balacheff(1987), Bell (1979), and Dormolen (1977, see Table 2). An algebraic adaptation of this framework was utilized to examine the justifications of students in this study. Similar to Simon and Blume, there are three levels: Level 0 (no justification), Level 1 (an appeal to external authority), and Level 2 (empirical demonstrations).

Table 2 Justification framework

Justification Level	Description
Level 0 : No justification	Responses do not address justification.
Level 1 : Appeal to external authority	Reference is made to the correctness stated by some other individual or reference material.
Level 2 : Empirical evidence	Justification is provided through the correctness of particular examples.
Level 3 : Generic example	Deductive justification is expressed in a particular instance.
Level 4 : Deductive justification	Validity is given through a deductive argument that is independent of particular instances.

Strategy

Strategies are an individual' s approach to a task. They are how a student organizes and uses a set of skill accomplish a particular task more effectively and efficiently. A description of these strategies and literature surrounding them is provided below.

Explicit strategy

Explicit reasoning has long been the focus of the algebra curricula (kaput, 1999). Explicit, or close-form, generalizations allow for the immediate calculation of any value for the particular situation by relating the independent variable to the dependent variable.

Recursive strategy

Recursive strategy is a rule that generates terms in a sequence through the preceding term or terms (NCTM, 2000). Recursive strategies are usually employed when a student knows a particular term and needs to find the next term or a value fairly close to it.

Whole-object Strategy

Whole-object reasoning as using a multiple of the output for a smaller input value to find the output for a larger input value(Stacey: 1989).

Chunking strategy

Chunking strategy is described as a recursive strategy where multiple "chunks" of the recursive value are added on to a known value.

Constructivism

The basic and most fundamental assumption of constructivism theory is that knowledge does not exist independent of the learner, knowledge is constructed. Within this theory falls two schools of thought, cognitive constructivism and social constructivism. Cognitive constructivism that is generally attributed to Jean Piaget, who articulated mechanisms by which knowledge is internalized by learners. He suggested that through processes of accommodation and assimilation, individuals construct new knowledge from their experiences. Lev Vygotsky is most often associated with the social constructivist theory. He emphasizes the influences of cultural and social contexts in learning.

How researcher developed the instructional process, show in Figure 1

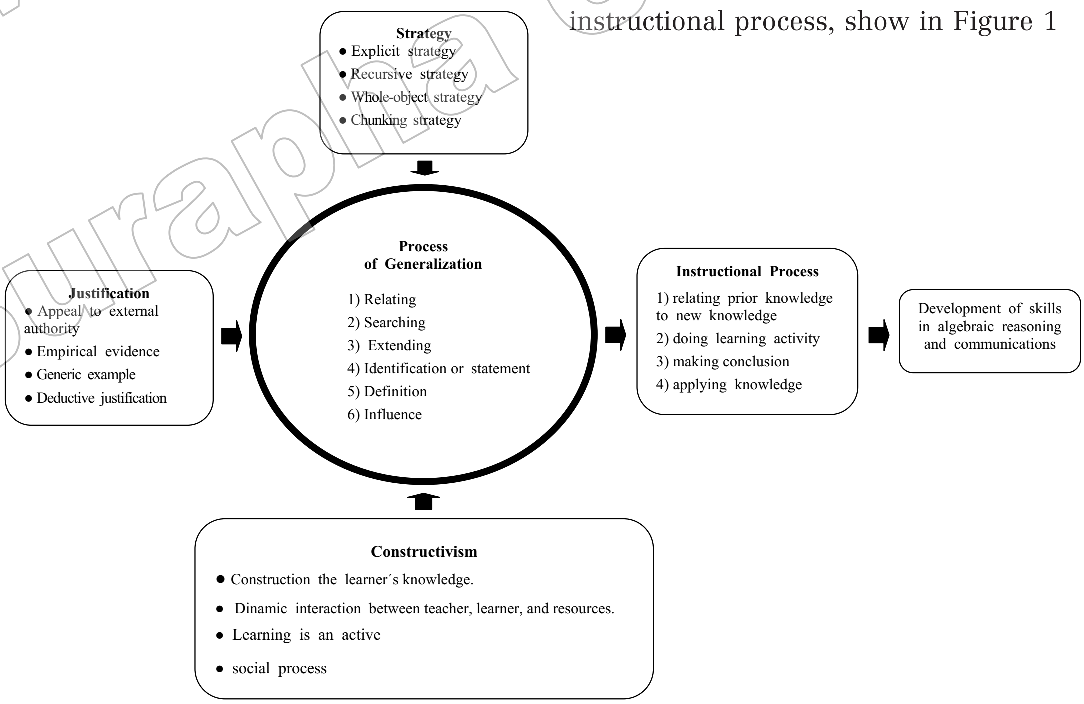

Figure 1. Conceptual Framework of a development of instructional process using the process of generalization

Objectives

The purposes of research were to :

1) develop instructional process by using process of generalization to enhance algebraic reasoning ability and mathematical communications of ninth grade students.

2) study the quality of the developed instructional process on algebraic reasoning ability and mathematical communications.

Methods

This research procedure was divided into two phases. The first phase was the development of the instructional process through analysis and synthesis the process of generalization, based on Ellis (2007) and integrated justification, strategy, and constructivism.

The second phase was the experiment of the developed instructional process to study the quality of instructional process using process of generalization to enhance algebraic reasoning ability and mathematical communication of ninth grade students. This phase focus on the development of research instrument for evaluating algebraic reasoning ability and mathematical communications, and instruction process evaluation. Instruments were: 1) lesson plans based on the process of instructional process by using process of generalization to enhance algebraic reasoning ability and mathematical communication developed by the researcher. 2) tests of algebraic reasoning ability developed by the researcher. 3) tests of mathematical communication developed by the researcher. Quality of Research Instruments were: 1) examine item content validity by 3 mathematics educators and selected only value of validity from item objective congruence (IOC) upper than 0.5. 2) examine objectivity by 3 mathematics educators in order to check the meaning of language. 3) examine level of difficulty and power of discrimination selected items with level of difficulty is in .20-.80 and power of discrimination is in .20-1.00 3) Reliability of each test use Cronbach Alpha efficient. The samples were 79 students, two classrooms which arithmetic mean of former mathematics achievement were not significantly higher, forms of ninth grade students at Piboonbumpen Demonstration School, Burapha University in 2009 academic year, and sample sampling into control and experimental group. The process was applied in 12 weeks. Data were analyzed by using arithmetic mean, standard deviation, and t-test.

Operational Definition

Instructional process using process of generalization was the developed instructional process consisted of 4 steps, namely: 1) relating prior knowledge to new knowledge 2) doing learning activity 3) making conclusion, and 4) applying knowledge.

Algebraic reasoning ability refers to students' ability in using their thinking to understand algebra. In this study, thinking that involves forming generalizations and drawing valid conclusions about ideas and how they are related.

Mathematical communication refers to students' ability in using words, numbers, symbols, and diagrams (or pictures) to gather, organize, and express mathematical ideas with clarity.

Findings

1. The developed instructional process of 4 steps. They are step1: relating prior knowledge to new knowledge, step 2: doing learning activity, step 3: making conclusion, and step 4: applying knowledge.

Step 1: Relating prior knowledge to new knowledge

The learning activities are as follows:

1. Teacher activates students' prior knowledge about the contents and concepts by using conflict situation.

2. Students recognize prior knowledge and transfer new knowledge.

Step 2: Doing learning activity

The learning activities are as follows:

1. Teacher designs problem situations which promote reasoning and communication of students.

2. Teacher enhances student acting self-directed-learning.

3. Teacher suggests the learning strategies for problem solving.

4. Student is an active learner participating and working in group.

5. Students observes the task for searching relationship / procedure / pattern / solution in order to detect a stable across all cases.

6. Students expand the range of relationship in order to generate new cases.

Step 3: Making conclusion

The learning activities are as follows:

1. Teacher facilitates students to experiment, practice, and making conclusion.

2. Teacher guides students to observe relationship / procedure / pattern / solution characteristic all cases.

3. Students create and explain a new pattern and rule.

Step 4: Applying knowledge.

The learning activities are as follows:

1. Teacher designs new problem situations that relates students learning experience.

2. Students adapt an existing generalization to apply to a new problem situation.

2. Algebraic reasoning and mathematical communications abilities of students in the experimental group after learning with developed instructional process were significantly higher than those of students in the control group at .05 level of significance.

Arithmetic means scores of post-test of algebraic reasoning and mathematical communication ability of students in experiment and control groups were compared with t-test and given in Table 1-2.

Table 1 experimental and control groups' t-test comparison of algebraic reasoning

Group	n	$\bar{X}$	SD	t
Experimental	40	28.20	2.03	9.587*
Control	39	22.15	3.39	

*p < .05

Table 2 experimental and control groups' t-test comparison of mathematical communication

Group	n	$\bar{X}$	SD	t
Experimental	40	28.93	2.02	9.399*
Control	39	23.03	3.38	

*p < .05

3. Algebraic reasoning and mathematical communications abilities of students in the experimental group after learning with developed instructional process were significantly higher than before learning with developed instructional process at .05 level of significance.

The pre-test and post-test score of algebraic reasoning and mathematical communication ability of students in experiment groups were compared with t-test and given in Table 3-4.

Table 3 t-test comparison of pre-test and post-test scores of algebraic reasoning of experiment groups.

Experimental Group	n	Pre-test		Post-test		t
		$\bar{X}$	SD	$\bar{X}$	SD	
Mathematical communication	40	12.08	2.89	28.20	2.03	48.793*

*p < .05

Table 4 t-test comparison of pre-test and post-test scores of mathematical communication of experiment groups.

Experimental Group	n	Pre-test		Post-test		t
		$\bar{X}$	SD	$\bar{X}$	SD	
Mathematical communication	40	13.30	2.85	28.93	2.02	35.423*

*p < .05

4. The finding of qualitative data found that the mathematical knowledge, algebraic reasoning and mathematical communication abilities of students in the experimental group were improved. They could be able to draw mathematical conclusions reasonably, and elaborate ideas by using mathematical language and symbols effectively.

Discussion

Algebraic reasoning and mathematical communication abilities of students in the experiment group were higher than those of students in the control group because of several causes. 1) Relating prior knowledge to new knowledge in the first step made students connect prior and new concepts, so they saw some relationship between contents or situations. 2) Doing learning activity by searching the relationship and expanding the range of cases so that they could see pattern and lead to generate general cases. 3) Students had opportunity to see relation of pattern and made conclusion by themselves. 4) Students usually used thinking process for a long time to make conclusion with themselves. This activity could develop their algebraic reasoning and mathematical communications abilities.

Although the results of this study revealed that the steps of the learning process could enable students to construct their knowledge. At the beginning of the experiment, the students could draw conclusion by using textbook, but they could not generalize for patterns or conclusions by themselves. This was because in traditional learning process students did not have opportunity to think or show their idea.

Guidelines for Application and Future Research

The results from the study are suggested some useful guidelines as the following:

For teaching and learning mathematics:

1) The four steps, namely: 1) relating prior knowledge to new knowledge 2) doing learning activity 3) making conclusion, and 4) applying knowledge enhance algebraic reasoning ability and mathematical communications.

2) The four steps in the instructional process should be considered the nature of the

students and the classroom atmosphere.

For future Research:

1) Development of instructional process by using process of generalization to enhance algebraic reasoning ability and mathematical communications should be conducted in different grade students both in primary and secondary school.

2) The four steps of instructional process should be used to promote problem solving, connection, and creative thinking skills.

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