ตัวแบบปริมาณการสั่งซื้อสินค้าแบบประหยัดที่มีสินค้าชำรุดภายใต้การซ่อมแซมสินค้าและการตรวจสอบสินค้าผิดพลาด

Economic Order Quantity Model for Imperfect Items Under Repair Option and Inspection Errors

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บทคัดย่อ

ในการศึกษาครั้งนี้ได้ขยายตัวแบบปริมาณการสั่งซื้อแบบประหยัดไปสู่กรณีที่มีสินค้าชำรุดในลอตของสินค้า โดยกำหนดให้มีกระบวนการตรวจสอบสินค้า 100% ในการตรวจสอบสินค้าอาจมีค่าคลาดเคลื่อนเกิดขึ้นถึงแม้ว่ามีการตรวจสอบสินค้า 100% จึงทำให้เกิดการคัดแยกสินค้าผิดพลาด ซึ่งแบ่งได้เป็นสองกรณี คือ กรณีที่ 1 คัดแยกสินค้าที่ไม่ชำรุดเป็นสินค้าชำรุด กรณีที่ 2 คัดแยกสินค้าชำรุดเป็นสินค้าที่ไม่ชำรุด นอกจากนี้สินค้าที่ชำรุดถูกคัดแยกเป็นสินค้าชำรุด จะถูกนำไปซ่อมแซมและนำกลับเข้าสู่คลังสินค้าเมื่อคลังสินค้าไม่มีสินค้าเหลืออยู่ การศึกษานี้ได้หานโยบายเหมาะสมที่สุดที่มีขนาดการสั่งซื้อสินค้าเหมาะสมที่สุด และผลกำไรรวมคาดหมายต่อหน่วยเวลาสูงสุด พร้อมทั้งมีการแสดงตัวอย่างเชิงตัวเลขและการวิเคราะห์ความไว

คำสำคัญ : ตัวแบบ EOQ สินค้าชำรุด ค่าคลาดเคลื่อนของการตรวจสอบ การคัดแยกผิดพลาด การซ่อมแซม

Abstract

In this study, an economic order quantity (EOQ) model is extended to the case of imperfect items in a product lot. A 100% inspection process of the lot is conducted. An inspection error can occur despite a 100% inspection. From which, the misclassification is occurred that consists of two cases: (i) classifying non-defective items as defective, (ii) classifying defective items as non-defective. Moreover, for classifying defective items as defective, these defective items are sent to repair and will be returned to inventory when items in inventory are empty. This study is to determine the optimal policy with optimal order size and maximum expected total profit per unit of time. A numerical example and sensitivity analysis are exemplified.

Keywords : EOQ model, imperfect item, inspection error, misclassification, repair

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Introduction

One assumption of classical economic order quantity (EOQ) models is that the ordered items are perfect. However, the ordered items may be defective due to deficient planned maintenance or weak production control. Many researchers have developed different economic order quantity models with imperfect items. Salameh and Jaber (2000) developed an EOQ model when items were not of a perfect quality. They assumed a 100% inspection process in which the defective quality items were withdrawn from inventory and sold as a single batch. This work has served as a fundamental basis for similar works of many other researchers in the field. Papachristos and Konstantaras (2006) proposed an EOQ model based on the work of Salameh and Jaber (2000). This model discussed about shortage avoidance when the defective items were withdrawn from inventory after the inspection process. Eroglu and Ozdemir (2007) developed an EOQ model with imperfect items and shortages backordered under 100% of each lot were screened. Wee *et al*. (2007) extended the work of Salameh and Jaber (2000) and assumed that shortage was allowed. Maddah and Jaber (2008) revisited the model of Salameh and Jaber (2000) to adjust a fault in the model of Salameh and Jaber. Jaber *et al*. (2008) extended the EOQ model of Salameh and Jaber (2000) by assuming the percentage of defective per lot reduced in agreement with the learning curve. Hsu and Yu (2009) investigated an EOQ model with imperfect items under a one-time-only sale when the imperfect items were screened out and sold as a single batch at the end of the 100% inspection process. Chang and Ho (2010) developed an EOQ model based on the work by Wee *et al*. (2007). This model applied the well-known renewal theorem to obtain a closed-form optimum solution. Lin (2010) developed a new inventory model for imperfect quality and quantity discounts where the buyer exerted power over its supplier. Lin assumed that the defective items were screened out by 100% inspection and sold as a single batch at the end of the cycle and also assumed that the order quantity was manufactured at on setup. Khan *et al*. (2010) extended a model of Salameh and Jaber (2000) with three different scenarios of learning inspection. This model considered situations of lost sales and backorders. Konstantaras *et al*. (2012) examined an EOQ model with imperfect item shortage under learning inspection. This model was developed for infinite and finite planning horizons. Liu and Zheng (2012) proposed an EOQ model with imperfect items, shortage under inspection errors, where the fraction of defective was assumed to be a fuzzy number. Hsu and Hsu (2013) developed an EOQ model with imperfect quality items, inspection errors, shortage backordering and sale returns. They also assumed that when a shortage was allowed, all customers were willing to wait for the next delivery. Jaber *et al*. (2014) extended the EOQ model of Salameh and Jaber (2000) and presented two models. In the first model, they assumed that defective items were sent to repair at the repair shop; whereas, for the second model, defective items were assumed to be sold as a single bath and replaced by emergency order quality. Paul *et al*. (2014) proposed an EOQ model for multiple items, where a certain percentage of defection for some items was given. They considered two cases of the ordering policy; with and without a price discount.

In this study, the EOQ model of Jaber and *et al*. (2014) is extended with inspection error. The inspection process is also imperfect due to misclassification in which the non-defective items will be classified as defective and defective items will be classified as non-defective. Also, the classified defective items are again screened and classified. The non-defective items are returned to inventory while the defective items are repaired and will be returned to inventory when the inventory is empty. The rest of the paper is organized as follows. Section 2 describes the inventory model and produces an optimal policy. Section 3 presents a numerical example and sensitivity analysis. Section 4 presents a conclusion of our study.

Inventory model

In every cycle with a length time T , a lot size y is delivered with a constant demand rate D . We assume that the lot size contains defective items ρy where ρ is a fraction of defective items. A 100% inspection process is conducted at a rate $x > D$ and defective items ρy will be withdrawn from inventory at the end of inspection time period t_1 . The inspection error can occur despite a 100% inspection. Thus, the misclassification consists of two cases: (i) classifying non-defective items as defective with probability m_1 , these items (B_1) will be returned to inventory at the end of time period t_2 after they have been screened before repairing, (ii) classifying defective items as non-defective with probability m_2 , these items will be sent to a customer. We also assume that the customer would not detect and return them to inventory. Moreover, the defective items (B_2), are sent to repair and they will be returned to inventory when items in inventory are empty at the end of time period t_3 . The behavior of inventory levels is illustrated in Figure 1.

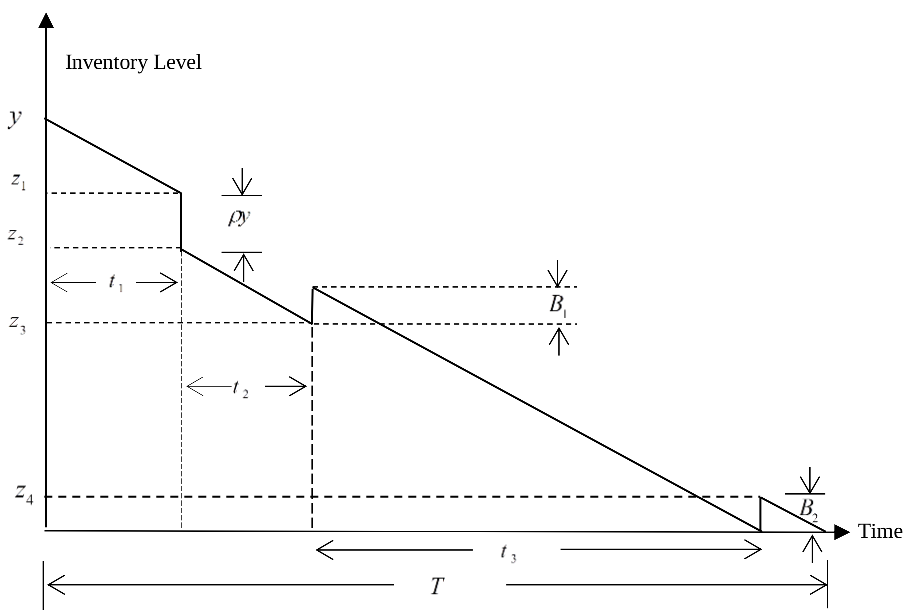

Figure 1 Inventory levels with inspection errors and repair option for defective items.

Notations

y	order size
D	demand rate
ρ	fraction of defective items
x	inspection rate
R	repair rate
t_1	time to screen a lot size y
t_2	time to screen a defective lot size ρy
t_3	time to repair defective items
t_T	total transport time of defective items to repair and return to inventory
T	cycle length time
K	ordering cost
S	repair setup cost
c_u	unit cost
h	holding cost per unit of non-defective items
h_R	holding cost per unit of repaired items
h'	holding cost per unit of repair facility
A	transportation fixed cost
c_m	unit material and labor cost
c_T	unit transportation cost
c_I	unit inspection cost
c_a	accepting a defective item cost
c_r	rejecting a non-defective item cost
s	unit selling price
m	markup percentage
m_1	probability of classifying non-defective items as defective
m_2	probability of classifying defective items as non-defective
$f(\rho)$	probability density function of ρ
$f(m_1)$	probability density function of m_1
$f(m_2)$	probability density function of m_2
$E[.]$	the expectation of a random variable
B_1	the number of non-defective items that are classified defective
B_2	the number of defective items that are returned after repaire

Consider a misclassification, a number of items are classified according to the misclassification from the screening of items. In such inspection process, there are four cases. Case (i) Non-defective items are classified as non-defective, $y(1-\rho)(1-m_1)$; Case (ii) Non-defective items are classified as defective, $y(1-\rho)m_1$; Case (iii) Defective items are classified as non-defective, $y\rho m_2$; Case (iv) Defective items are classified as defective, $y\rho(1-m_2)$ where $B_1 = y(1-\rho)m_1$, $B_2 = y\rho(1-m_2)$, $t_1 = \frac{y}{x}$, $t_2 = \frac{\rho y}{x}$, $t_3 = \frac{B_2}{R} + t_T$, $z_1 = y - Dt_1$, $z_2 = y(1-\rho) - y\rho m_2 - Dt_1$, $z_3 = y(1-\rho) - y\rho m_2 - D(t_1+t_2)$ and $z_4 = y\rho(1-m_2) - D(t_1+t_2+t_3)$.

To avoid shortage when the defective items will be withdrawn from the inventory at the end of the inspection time period, we assume that the numbers of non-defective items are equal and greater than demand rate of customers. Thus,

$$y - y(1-\rho)m_1 - y\rho(1-m_2) \geq DT$$

$$y - (ym_1 - y\rho m_1) - (y\rho - y\rho m_2) \geq DT$$

$$y - ym_1 + y\rho m_1 - y\rho + y\rho m_2 \geq DT$$

$$y(1-m_1) - y\rho(1-m_1) + y\rho m_2 \geq DT$$

$$y(1-m_1)(1-\rho) + y\rho m_2 \geq DT.$$

Without loss of generality, the cycle length time is given as

$$T = \frac{y(1-m_1)(1-\rho) + y\rho m_2}{D}. \quad (1)$$

Consider the different costs in this inventory system, first of all, the order cost (OC) is given as

$$OC = K + c_u y. \quad (2)$$

The inspection cost (IC) per cycle is a summation of the inspection cost of a lot size per units, inspection cost of defective items per units and cost of misclassification, it is given as

$$IC = c_I y + c_I y\rho + c_r(1-\rho)ym_1 + c_a \rho y m_2. \quad (3)$$

The repairing cost (RC) is given as

$$RC = (1+m)\left(S + 2A + y\rho(1-m_2)\left(c_m + 2c_T + h' t_3\right)\right). \quad (4)$$

The holding cost (HC) is given as

$$HC = h\left\{\frac{y^2\rho(1-m_2)}{x} + \frac{y^2(1-\rho)^2(1-m_1^2)}{2D} + \frac{y^2(\rho-\rho^2)m_2}{D} + \frac{y^2(1-\rho)^2 m_1^2}{D}\right\} + h_R\left\{\frac{y^2\rho^2(1-m_2)^2}{2D}\right\}. \tag{5}$$

So, the total cost (TC) per cycle is given by

$$\begin{aligned} TC &= OC + IC + RC + HC \\ &= K + c_u y + c_I y + c_I y\rho + c_r(1-\rho)ym_1 + c_a\rho ym_2 + (1+m)\left(S + 2A + y\rho(1-m_2)(c_m + 2c_T + h' t_3)\right) \\ &\quad + h\left\{\frac{y^2\rho(1-m_2)}{x} + \frac{y^2(1-\rho)^2(1-m_1^2)}{2D} + \frac{y^2(\rho-\rho^2)m_2}{D} + \frac{y^2(1-\rho)^2 m_1^2}{D}\right\} + h_R\left\{\frac{y^2\rho^2(1-m_2)^2}{2D}\right\}. \end{aligned} \tag{6}$$

The total revenue (TR) per cycle is the sum of total sale of non-defective items and total sale of repaired items,

$$TR = s\left(y(1-\rho)(1-m_1) + y\rho m_2\right) + sy(1-\rho)m_1 + sy\rho(1-m_2). \tag{7}$$

The total profit (TP) per cycle is a difference between the total revenue (TR) per cycle and the total cost (TC) per cycle. It is given as

$$\begin{aligned} TP &= TR - TC \\ &= s\left(y(1-\rho)(1-m_1) + y\rho m_2\right) + sy(1-\rho)m_1 + sy\rho(1-m_2) - K - c_u y - c_I y - c_I y\rho \\ &\quad - c_r(1-\rho)ym_1 - c_a\rho ym_2 - (1+m)\left(S + 2A + y\rho(1-m_2)(c_m + 2c_T + h' t_3)\right) \\ &\quad - h\left\{\frac{y^2\rho(1-m_2)}{x} + \frac{y^2(1-\rho)^2(1-m_1^2)}{2D} + \frac{y^2(\rho-\rho^2)m_2}{D} + \frac{y^2(1-\rho)^2 m_1^2}{D}\right\} - h_R\left\{\frac{y^2\rho^2(1-m_2)^2}{2D}\right\} \\ &= s\left(y(1-\rho)(1-m_1) + y\rho m_2\right) + sy(1-\rho)m_1 + sy\rho(1-m_2) - K - c_u y - c_I y - c_I y\rho \\ &\quad - c_r(1-\rho)ym_1 - c_a\rho ym_2 - (1+m)(S + 2A) - (1+m)\left(y\rho(1-m_2)(c_m + 2c_T + h' t_T)\right) \\ &\quad - (1+m)\left(\frac{y^2\rho^2(1-2m_2+m_2^2)}{R}\right) \\ &\quad - h\left\{\frac{y^2\rho(1-m_2)}{x} + \frac{y^2(1-\rho)^2(1-m_1^2)}{2D} + \frac{y^2(\rho-\rho^2)m_2}{D} + \frac{y^2(1-\rho)^2 m_1^2}{D}\right\} - h_R\left\{\frac{y^2\rho^2(1-m_2)^2}{2D}\right\} \end{aligned} \tag{8}$$

where $t_3 = \frac{y\rho(1-m_2)}{R} + t_T$.

Since ρ, m_1 and m_2 are the random variables with probability density function $f(\rho)$, $f(m_1)$ and $f(m_2)$, respectively, the expected total profit per cycle is given as

$$E(TP) = s\left(y\left(1-E(\rho)\right)\left(1-E(m_1)\right) + yE(\rho)E(m_2)\right) + sy\left(1-E(\rho)\right)E(m_1) + syE(\rho)\left(1-E(m_2)\right)$$
$$-K - c_u y - c_I y - c_I yE(\rho) - c_r y(1-E(\rho))E(m_1) - c_a yE(\rho)E(m_2) - (1+m)(S+2A)$$
$$-(1+m)\left(yE(\rho)\left(1-E(m_2)\right)(c_m + 2c_T + h't_T)\right) - (1+m)\left(\frac{y^2E(\rho^2)\left(1-2E(m_2)+E(m_2^2)\right)h'}{R}\right)$$
$$-h\left\{\frac{y^2E(\rho)(1-E(m_2))}{x} + \frac{y^2\left(1-2E(\rho)+E(\rho^2)\right)\left(1-E(m_1^2)\right)}{2D}\right.$$
$$\left. + \frac{y^2\left(E(\rho)-E(\rho^2)\right)E(m_2)}{D} + \frac{y^2\left(1-2E(\rho)+E(\rho^2)\right)E(m_1^2)}{D}\right\}$$
$$-h_R\left\{\frac{y^2E(\rho^2)\left(1-2E(m_2)+E(m_2^2)\right)}{2D}\right\}. \tag{9}$$

From Eq. (1), the expected cycle length time is given as

$$E(T) = \frac{y(1-E(m_1))(1-E(\rho)) + yE(\rho)E(m_2)}{D}. \tag{10}$$

when ρ and m_1 are independent of each other, and also, ρ and m_2 are independent of each other.

Using the renewal reward theorem (Maddah and Jaber, 2008), the expected total profit per unit of time can be written as

$$E(TPU) = \frac{E(TP)}{E(T)}$$
$$= sD + \frac{D\left[s(1-E(\rho))E(m_1) + sE(\rho)(1-E(m_2))\right]}{(1-E(m_1))(1-E(\rho)) + E(\rho)E(m_2)}$$
$$-\frac{D}{(1-E(m_1))(1-E(\rho)) + E(\rho)E(m_2)}\left[\frac{K}{y} + c_u + c_I + c_I E(\rho) + c_r(1-E(\rho))E(m_1)\right.$$
$$+c_a E(\rho)E(m_2) + \frac{(1+m)(S+2A)}{y} + (1+m)\left(E(\rho)(1-E(m_2))(c_m + 2c_T + h't_T)\right)$$
$$+(1+m)\left(\frac{yE(\rho^2)\left(1-2E(m_2)+E(m_2^2)\right)h'}{R}\right) + hy\left\{\frac{E(\rho)(1-E(m_2))}{x}\right.$$
$$+\frac{\left(1-2E(\rho)+E(\rho^2)\right)\left(1-E(m_1^2)\right)}{2D} + \frac{\left(E(\rho)-E(\rho^2)\right)E(m_2)}{D}$$
$$\left.\left. + \frac{\left(1-2E(\rho)+E(\rho^2)\right)E(m_1^2)}{D}\right\} + h_R y\left\{\frac{E(\rho^2)\left(1-2E(m_2)+E(m_2^2)\right)}{2D}\right\}\right]. \tag{11}$$

The optimal value $y*$ can be obtained by minimizing $E(TPU)$ of Eq. (11). Setting the first derivative of $E(TPU)$ with respect to y equal to zero and solving yields the solution

$$y^* = \left(\frac{2D\left(K + (1+m)(S+2A)\right)}{h\left\{ \frac{2DE(\rho)(1-E(m_2))}{x} + F_1\left(1-E\left(m_1^2\right)\right) + 2F_2 + 2F_1E\left(m_1^2\right)\right\} + 2(1+m)\left(h'D\frac{E(\rho^2)F_3}{R}\right) + h_R\left(E(\rho^2)F_3\right)} \right)^{\frac{1}{2}} \quad (12)$$

where $F_1 = 1 - 2E(\rho) + E\left(\rho^2\right)$, $F_2 = \left(E(\rho) - E\left(\rho^2\right)\right)E(m_2)$ and $F_3 = 1 - 2E(m_2) + E\left(m_2^2\right)$.

Note that when there is no misclassification ($m_1 = m_2 = 0$), the optimal order size reduces to the EOQ of Jaber *et al*. (2014) formula.

Taking the second derivative of $E(TPU)$ with respect to y, we have

$$\frac{d^2}{dy^2}E(TPU) = -\frac{D}{(1-E(m_1))(1-E(\rho)) + E(\rho)E(m_2)}\left\{\frac{K + (1+m)(S+2A)}{y^3}\right\}. \quad (13)$$

Since $D > 0$, $0 < E(\rho) < 1$, $0 < E(m_1) < 1$ and $0 < E(m_2) < 1$ for every $y > 0$, then $\frac{d^2}{dy^2}E(TPU) < 0$ which implies that there exists a unique value of y that maximizes Eq.(11).

Numerical example and sensitivity analysis

In this section, the values of the parameter are adopted from Khan *et al*. (2011) and Jaber *et al*. (2014). These values are shown as follows.

D	50,000 unit/year
x	175,200 unit/year
R	50,000 unit/year
s	$50/unit
K	$100
S	$100
c_u	$25/unit
h	$5/unit
h_R	$6/unit
h'	$4/unit
c_I	$0.5/unit
A	$200
c_T	$2/unit

c_m $5/unit

c_a $500/unit

c_r $100/unit

t_T 2/200 year

m 0.2

The fraction of defective items (ρ), the probability of classifying non-defective items as defective (m_1) and the probability of classifying defective items as non-defective (m_2) has a uniform distribution in $(0, \theta)$, $(0, \beta)$ and $(0, \lambda)$, respectively where the upper bound of ρ , m_1 and m_2 are $\theta = \beta = \lambda = 0.05$. The probability density function of ρ , m_1 and m_2 are given as

$$f(\rho) = \begin{cases} 20, & 0 < \rho < 0.05 \\ 0, & \text{otherwise} \end{cases}$$

$$f(m_1) = \begin{cases} 20, & 0 < m_1 < 0.05 \\ 0, & \text{otherwise} \end{cases}$$

$$f(m_2) = \begin{cases} 20, & 0 < m_2 < 0.05 \\ 0, & \text{otherwise} \end{cases}$$

Then, we have

$$E(\rho) = \int_0^{0.05} \rho f(\rho)\, d\rho = 0.025$$

$$E(m_1) = \int_0^{0.05} m_1 f(m_1)\, dm_1 = 0.025$$

$$E(m_2) = \int_0^{0.05} m_2 f(m_2)\, dm_2 = 0.025$$

$$E\left(\rho^2\right) = \int_0^{0.05} \rho^2 f(\rho)\, d\rho = 0.00083$$

$$E\left(m_1^2\right) = \int_0^{0.05} m_1^2 f(m_1)\, dm_1 = 0.00083$$

$$E\left(m_2^2\right) = \int_0^{0.05} m_2^2 f(m_2)\, dm_2 = 0.00083.$$

We substitute the above values of the parameter into Eq. (10), Eq. (12) and Eq. (11), the optimum solution is T = 0.0723 years, y^* = 3,800.61 units and $E(TPU)$ = $1,038,250.83/year, respectively. Also, Fig. 2 shows that the expected total profit per unit of time is concave in order size y and the optimal order size y^* which maximizes the expected total profit per unit of time.

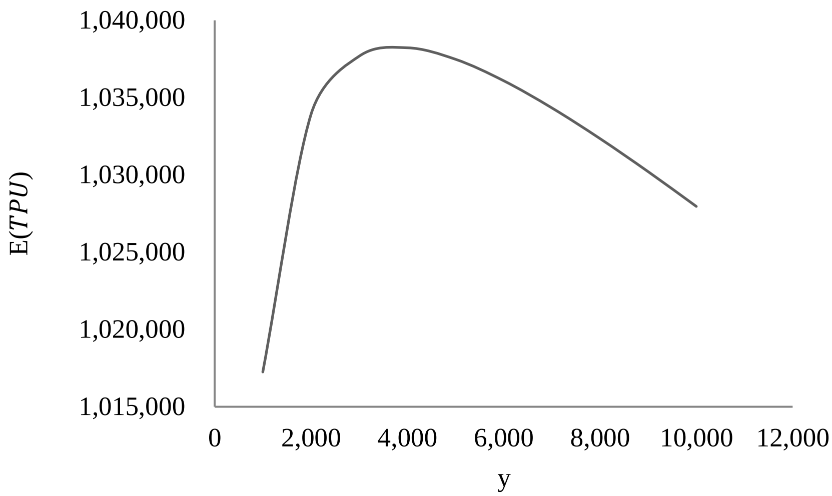

Figure 2 Expected total profit per unit of time and optimal the order size

For sensitivity analysis, we consider behavior of the expected total profit per unit of time by considering in cases of (i) θ = 0.01, 0.02, 0.03, 0.04, 0.05 (ii) β = 0.01, 0.02, 0.03, 0.04, 0.05 and (iii) λ = 0.01, 0.02, 0.03, 0.04, 0.05 which are shown in Tables 1 – 3, respectively.

Table 1 $E(TPU)$ when ρ is uniformly distributed in $(0, \theta)$, where β = 0.05 and λ = 0.05

θ	y^*	$E(TPU)$
0.01	3,752.96	1,065,839.59
0.02	3,765.45	1,059,144.95
0.03	3,777.56	1,052,319.17
0.04	3,789.29	1,045,355.52
0.05	3,800.61	1,038,250.83

Table 1 shows that the upper bound of ρ is increasing at fixed β = 0.05 and λ = 0.05, the optimal order size is increasing and the expected total profit per unit of time is decreasing. Because the non-defective items are used to fulfill the demand of customer when the fraction of the defective items in the order lot is increasing, the order size will be large. However, the expected total profit per unit of time is decreasing, because there occurs an additional cost of repairing when the order size has many defective items in the lot.

Table 2 $E(TPU)$ when m_1 is uniformly distributed in $(0, \beta)$, where θ = 0.05 and λ = 0.05

β	y^*	$E(TPU)$
0.01	3,802.10	1,148,858.42
0.02	3,801.91	1,122,051.61
0.03	3,801.60	1,094,693.00
0.04	3,801.17	1,066,765.38
0.05	3,800.61	1,038,250.83

Table 3 $E(TPU)$ when m_2 is uniformly distributed in $(0, \lambda)$, where θ = 0.05 and β = 0.05

λ	y^*	$E(TPU)$
0.01	3,801.74	1,049,946.09
0.02	3,801.46	1,047,017.45
0.03	3,801.18	1,044,092.02
0.04	3,800.89	1,041,169.82
0.05	3,800.61	1,038,250.83

Table 2 shows the effect of probability of classifying non-defective items as defective at fixed θ = 0.05 and λ = 0.05, when the upper limit distribution is increasing, and the optimum order size and expected total profit per unit of time are decreasing. This effect is not significant because when the non-defective items classified as defective are be returned to inventory, it depends on the optimum order size. However, the expected total profit per unit of time is decreasing because of the cost of rejecting the non-defective items.

Table 3 shows that the effect of probability of classifying defective items as non-defective at fixed θ = 0.05 and β = 0.05 has a similar effect as the probability of classifying non-defective items as defective. When the upper limit probability of classifying defective items as non-defective is increasing, the optimum order size and the expected total profit per unit of time are decreasing.

Conclusion

In this paper, we develop an economic order quantity (EOQ) model with repairing under imperfect items and 100% inspection process. An inspection error can occur despite a 100% inspection, so, the misclassification would consist of two cases that are (i) classifying non-defective items as defective and (ii) classifying defective items as non-defective. For classifying defective items as defective, they are sent to repair and will be returned to inventory when items in inventory are empty. The expected total profit per unit of time is derived. We have found that the expected total profit per unit of time remains concave with respect to the order size. Moreover, we have obtained the optimal order size and time for replenishment.

Sensitivity analysis shows that the expected total profit per unit of time is decreasing in ρ , m_1 and m_2 . Further research can be conducted to consider stochastic demand.

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